The old and new Concepts of Physics
An open-dialogue journal

www.uni.lodz.pl/concepts

ANTIMATTER AND PSEUDOTACHYONIC RELATIVITY

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(Received 27 June 2006; revised 27 February 2007; accepted 15 May 2007)

Abstract

A new conception of *antiparticle* is proposed as the subluminal“image” of a tachyonic homologous particle. After a first article on the subject, we begin by studying the pseudotachyonic transformation for energy, linear momentum and mass. We'll finally conclude that:

- antiparticles must have negative energy (and massive ones also negative masses) and opposite electric charge;
- antiphotons behave differently from photons;
- this difference appears, for instance, in the Compton effect generalized to antiparticles;
- the annihilation of colliding homologous particles and pair creation through colliding particles (this one yet an open problem) doesn't exactly fit in its standard description;

DOI: 10.2478/v10005-007-0024-9

- antimatter is characterized by negative absolute temperatures and the inversion of the 2nd law of Thermodynamics.

1 Introduction: Tachyons and antiparticles

In a former article [1] I proposed a reinterpretation of standard Lorentz transformations for $|v| > c$ — let's say "Lorentz tachyonic transformations" — and subsequently the fundaments of a Pseudotachyonic Relativity, particularly in what concerns the description of motion. According to this theory, the **principle of equivalence** must be extended to **all** reference frames, bradyonic or tachyonic. However, reciprocally tachyonic frames cannot directly detect one another — this physical impossibility being mathematically translated by imaginary numbers for measurable variables. This means, in particular, that a tachyon may never be directly detected.

Instead, a tachyonic frame S" may only be detected as its "associated frame", said "pseudotachyonic", obtained from the first by an interchange of space-time axis and which has lower-than-light velocity:

$$\hat{v} = c^2/v;$$

this velocity $\hat{v}$ (read "hat v") is called "associated velocity" or "covelocity" of v. The mentioned **pseudotachyonic frame** is symbolised by S^* and the correspondent "Lorentz pseudotachyonic transformations", for $\beta = \frac{v}{c} > 1$, are

$$\begin{cases} x^* = \frac{x.\beta - c.t}{\alpha} \\ y^* = y \\ z^* = z \\ t^* = \frac{x/c - \beta.t}{\alpha} \end{cases} \quad \text{in which } \alpha = \sqrt{\beta^2 - 1}. \tag{1}$$

As usual in pseudotachyonic transformations, the inverse transformation (from S^* to S) is absolutely identical to the former:

$$\begin{cases} x = \frac{x^*.\beta - c.t^*}{\alpha} \\ y = y^* \\ z = z^* \\ t = \frac{x^*/c - \beta.t^*}{\alpha}. \end{cases} \tag{1.a}$$

This frame S^* behaves **almost** like an ordinary subluminal frame. However, remarkably, **time appears in an inverted flow**. For instance, if we consider the usual bradyonic frame *S'* which velocity is $u = c^2/v$, equal to that of S^* (resulting $\beta_u = u/c = 1/\beta$), we may easily conclude that, transforming the quadrivector $(x,\, y,\, z,\, ict)$ into both frames,

$$x^* = x', \quad y^* = y', \quad z^* = z', \quad t^* = -t'.$$

This ordinary bradyonic frame, *S'*, is named **paraframe** of S^*. It is quite useful for a better understanding and reasoning because it allows a direct comparison between bradyonic and pseudotachyonic transformations concerning any physical variable.

"As mentioned before, this mathematical solution — *in avoiding imaginary values for the measurable variables of space and time — is also a consistent* physical solution *to the problem of tachyonic reference frames. These new proposed transformations and others (derived from them or related to them) shall describe not exactly **tachyons** – which cannot be directly detected — but their **"image"** in a subluminal reference system; and this "image" is the only way we can* "see" *them"* [1].

We'll attack now the major problems involved in such detection. We'll conclude that, as I pointed out in my precedent article, tachyons are very physical; they have indeed *"been observed in nature: in fact, they can only appear to us in the form of* subluminal — *also said* bradyonic — antiparticles*, and by this I take the liberty of meaning particles not only of opposite charge but also with* negative energy*"*. This includes not only massive antiparticles but non-massive ones, like photons.

2 Energy, linear momentum and mass

In tensorial notation, equally to the space-time quadrivector $x^a = (x,\, y,\, z,\, ict)$, any generic contravariant quadrivector $\mathbf{A}^a$ transforms according to the **bradyonic law** [2]:

$$\begin{cases} \mathbf{A}'^1 = \frac{\mathbf{A}^1 + i.\beta.\mathbf{A}^4}{\sqrt{1-\beta^2}} \\ \mathbf{A}'^2 = \mathbf{A}^2 \\ \mathbf{A}'^3 = \mathbf{A}^3 \\ \mathbf{A}'^4 = \frac{\mathbf{A}^4 - i.\beta.\mathbf{A}^1}{\sqrt{1-\beta^2}}. \end{cases} \tag{2}$$

As explained in [1], we must accept that these transformations also apply to any S'' tachyonic. But then (for $\beta > 1$), if S^* is its associated frame, we'll have the symmetrical condition [equivalent to the first equations (2), subsection 1.2]:

$$\begin{cases} \mathbf{A}^{*1} = -\mathbf{A}''^{4} \\ \mathbf{A}^{*4} = -\mathbf{A}''^{1}; \end{cases} \tag{3}$$

therefore, the **pseudotachyonic transformation law** for the quadrivector $\mathbf{A}^a$ is:

$$\begin{cases} \mathbf{A}^{*1} = \frac{i.\mathbf{A}^4+\beta.\mathbf{A}^1}{\alpha} \\ \mathbf{A}^{*2} = \mathbf{A}^2 \\ \mathbf{A}^{*3} = \mathbf{A}^3 \\ \mathbf{A}^{*4} = \frac{i.\mathbf{A}^1-\beta.\mathbf{A}^4}{\alpha}. \end{cases} \tag{3.a}$$

If we apply this law to the space-time quadrivector, we'll obtain the four equations (1). Now, we'll apply it to the transformation of the *linear momentum-energy* quadrivector (p_x, p_y, p_z, $i.E/c$), as well as the conditions (3), which in this case correspond to (from now on, we'll consider $\beta > 1$; for $\beta < -1$ consult Appendix A in [1])

$$\begin{cases} p_x^* = -i.c.\left(E''/c^2\right) \\ i.c.\left(E^*/c^2\right) = -p_x''; \end{cases} \tag{4}$$

the resulting momentum-energy transformation law is:

$$\begin{cases} p_x^* = \frac{p_x.\beta-E/c}{\alpha} \\ p_y^* = p_y \\ p_z^* = p_z \\ E^* = \frac{p_x.c-E.\beta}{\alpha}. \end{cases} \tag{5}$$

If, in particular, the movement is parallel to the xx axis, the equation system reduces to

$$\begin{cases} p^* = \frac{p.\beta-E/c}{\alpha} \\ E^* = \frac{p.c-E.\beta}{\alpha}, \end{cases} \quad \text{and, identically,} \quad \begin{cases} p = \frac{p^*.\beta-E^*/c}{\alpha} \\ E = \frac{p^*.c-E^*.\beta}{\alpha}. \end{cases} \tag{5.a}$$

Considering now $p^* = p_0^* = 0$, we obtain the linear momentum p and the energy E for a tachyon's "image" in S as a function of its **rest energy** (measured in S^*):

$$p = -\frac{E_0^*}{c.\alpha} \quad \text{and} \quad E = -\frac{\beta}{\alpha}.E_0^* \,. \tag{5.b}$$

The inverse transformations, for $p = p_0 = 0$, are quite identical:

$$p^* = -\frac{E_0}{c.\alpha} \quad \text{and} \quad E^* = -\frac{\beta}{\alpha}.E_0 \,. \tag{5.c}$$

¿From (5) it's easy to conclude that, in general, if *S'* is the paraframe of *S**, then

$$\begin{cases} \mathbf{p}^* = \mathbf{p}' \\ E^* = -E' \,. \end{cases}$$

This is certainly a result one should expect in virtue of the correspondence $(x,\, y,\, z,\, ict) \Leftrightarrow (p_x,\, p_y,\, p_z,\, i.E/c)$. It has nevertheless an extraordinary meaning: *a **tachyonic particle** with velocity v will be detected with a **subluminal velocity** $\hat{v} = c^2/v$ but with a **negative energy!***

This offers us a path to study **antimatter** from a strictly relativistic point of view. But I must signalise the very instant a first and fundamental difference in the concept of *antiparticle* with regard to Quantum Theory. In a way it recovers, despite several discrepancies, an element of Dirac's original concept; however, in his idea an antiparticle shouldn't have negative but *positive* energy and opposite charge, corresponding, for instance, an anti-electron to a 'missing electron' with negative energy. We'll see in section 4 that these opposite charges arise naturally in Pseudotachyonic Relativity. I propose, then, that *an **antiparticle** must be simply a particle with **negative energy**.*

A first consequence, of cosmological relevance, is that there is no need of calling upon a third kind of matter, in the category of *"exotic matter"*, whose energy is supposed to be negative, contrary to that of *antimatter*, supposed positive. According to the present theory, there is a sort of 'double mirror' (the speed of light) and both sides of it (related to matter and antimatter) which mutual relationship is perfectly reflexive. This seems enough. In conclusion, such "negative exotic matter" must be, in fact, antimatter.

Actually, we don't have to worry about the problem that led Dirac subsequently to interpret an anti-electron as a gap in a boundless sea of negative energy, in which usually almost all the states are occupied. This conception is hard to accept and Quantum Physics have learned to go round it. The point is that, in fact, the energetic *zero* appears, for a particle, as an unsurpassable barrier (just like the speed of light or the zero in the scale of absolute temperatures). This means that any particle (massive or not) has either a positive or a negative energy; it cannot naturally transform one in the other, these energies being like realities on both sides of a 'double mirror' that cannot be ordinarily crossed. So the *lowest energy* for a particle is the one nearest zero — its *proper energy* (see next section) —, and this on *both sides* of the 'mirror'.

Remark that the invariant

$$p_x^2 + p_y^2 + p_z^2 - E^2/c^2 = -m_0^2.c^2$$

presupposed in the transformation of the quadrivector (p_x, p_y, p_z, $i.E/c$) remains obviously valid in Pseudotachyonic Relativity. It conduces to the equation

$$E = \pm c.\sqrt{m_0^2.c^2 + p^2},$$

the – sign applying to antiparticles.

On the other hand, *a* ***negative energy*** *must correspond to a* ***negative mass***. In fact, if *S'* is the paraframe of *S**, since $E^* = -E'$, we'll have

$$E_0^* = -m_0.c^2\,;$$

hence, in order to preserve Einstein's equation $E = m.c^2$ in **all** frames, we must consider that $m_0 = -m_0^*$. We may write, then, $E_0^* = m_0^*.c^2$ and also $E^* = m^*.c^2$.

By the way, combining Einstein's equation with $p = m.v$, we obtain this one (also valid for non-massive particles, such as photons):

$$E = p.\hat{v}\,; \tag{6}$$

(of course, $\hat{c} = c$, and so the equation $E = p.c$ is valid for photons).

Finally, from these considerations and equations (5,a) we obtain the following relation for the measure of a *mass* in the frames S and

S^*:

$$m = -m^*.\frac{\beta - u^*/c}{\alpha}\,. \tag{7}$$

We are dealing here with a **negative mass** which, in modulus, is exactly the same measured in the paraframe of S^*. Making $u^* = 0$, it results the expression of m as a function of the *rest mass* m_0^* in the frame S^*:

$$m = -\tfrac{\beta}{\alpha}.m_0^*\,, \quad \text{or} \quad m = -\tfrac{m_0^*}{\sqrt{1-(1/\beta)^2}}\,. \tag{7.a}$$

Intuitively, it's not difficult to assume that a negative mass/energy must have an important consequence in what comes to *repulsive gravity*. In fact, it does — but this is beyond the scope of the present article.

3 Antiparticles

3.1 Massive antiparticles

Since we cannot detect a tachyonic frame as itself but only as its associated pseudotachyonic frame, *we also cannot detect a* ***tachyonic particle*** *as itself but only as its "image", its* ***pseudotachyonic associated****, and this detection corresponds to an* ***homologous antiparticle*** *moving with subluminal velocity* $\hat{v} = c^2/v$.

Consider a massive particle $\mathbf{P}$ moving along the xx axis of an arbitrary frame S with a tachyonic velocity v. If we suppose $\mathbf{P}$ to be at the origin of its own tachyonic frame S", it comes from what have been exposed that, in the frame S, the detection of $\mathbf{P}$ will be at the origin of the S" associated, S^*. We'll call the result of this detection in S the **antiparticle** of $\mathbf{P}$ (traditionally represented by $\overline{\mathbf{P}}$). It follows, as an immediate general conclusion, that every kind of particle shall possess an *homologous antiparticle*; this one will be no more than the detection of that sort of particle moving faster-than-light. Another important consequence is that **there is no difference in the nature of a particle and its homologous one, they are of the same kind**; the difference is nothing but a **relativistic effect** due to its state of movement relatively to the reference frame that evaluates it.

As we have seen before, both mass and energy of $\overline{\mathbf{P}}$ are negative. We may also conclude that, because of its negative energy, antiparticles have this remarkable dynamic characteristic: *the linear momentum vector* $\mathbf{p}$ *and the velocity vector* $\mathbf{u}$ *have opposite orientations* [see figure 1].

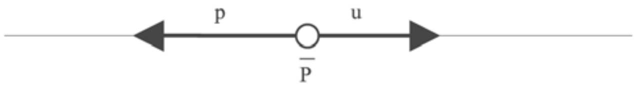

Figure 1: Velocity and momentum vectors for an antiparticle

Now, if we search the expression for the kinetic energy of this antiparticle $\overline{\mathbf{P}}$, which (subluminal) velocity is $u = \hat{v}$,

$$\overline{E_k} = \int_0^u F_T.ds = \int_0^u \frac{d}{dt}(m.u).ds\,,$$

we obtain, applying (7.a) and resolving the integral,

$$\overline{E_k} = \left(1 - \frac{1}{\sqrt{1-(u/c)^2}}\right).m_0^*.c^2\,. \tag{8}$$

This is a ***negative*** *kinetic energy.* We can write (8) as $\overline{E_k} = \overline{E_t} + m_0^*.c^2$, considering $\overline{E_t} = \overline{m}.c^2$ the total energy of $\overline{\mathbf{P}}$ and $\overline{m}$ its mass. If we now suppose $u = 0$, that is to say $\overline{E_k} = 0$, we obtain the expression for the *rest energy* (in relation to S), or the *proper energy*, of the antiparticle $\overline{\mathbf{P}}$:

$$\overline{E_0} = -m_0^*.c^2 = -E_0^*, \tag{9}$$

E_0^* being the rest energy (in relation to S^*) of the particle $\mathbf{P}$.

On the other hand, $\overline{E_0} = \overline{m_0}.c^2$ and so

$$\overline{m_0} = -m_0^*; \tag{9.a}$$

thus, we preserve the usual equation, writing

$$\overline{E_k} = \overline{E_t} - \overline{E_0} = \left(\overline{m} - \overline{m_0}\right).c^2. \tag{8.a}$$

Intrinsically, this means that the proper energy and the proper mass of a *particle* and of its *homologous antiparticle* have positive values for the first and negative for the second but exactly equals in modulus. In fact, it's easy to conclude that this rule applies to every pair of massive homologous particles moving with the same velocity.

It is opportune to remark that (according to the pseudotachyonic transformations for velocity, mass and energy) a *particle* $\mathbf{Q}$, bradyonic in the $\mathbf{P}$ associated frame, S^*, will appear in the frame S as an *antiparticle* $\overline{\mathbf{Q}}$; simultaneously, an *antiparticle* $\overline{\mathbf{Q}}$ in S^* will correspond to a *particle* $\mathbf{Q}$ in S. As pointed out before, the fact that a particle behaves or not like an antiparticle depends only on its state of movement relatively to a certain reference frame. Under this point of view, it seems there are two fundamental 'material physical realities' – or, should we say, two 'physical aspects of Reality' –, those of two mutually tachyonic frames. A third 'physical reality' should be the one of a 'photonic frame', in which space-time doesn't have any meaning and which is a kind of *zero*, an intermediate reality between the two first ones.

Let's examine now what happens if an antiparticle $\overline{\mathbf{P}}$ starts moving submitted to a *constant force* $\mathbf{F}$. According to equations (5.b) and (9.a), making subluminal velocity $u = \hat{v}$,

$$F = \frac{dp}{dt} = \frac{d}{dt}\left(-\frac{m_0^*.u}{\sqrt{1-(u/c)^2}}\right) = \frac{d}{dt}\left(\frac{\overline{m_0}.u}{\sqrt{1-(u/c)^2}}\right);$$

integrating this expression, considering F constant (and also $u = 0$ for $t = 0$), it results

$$F.t = \frac{\overline{m_0}.u}{\sqrt{1-(u/c)^2}}; \tag{10}$$

now, resolving this in relation to the velocity u, we obtain

$$u = \pm c.\frac{(F/\overline{m_0}.c)\,.t}{\sqrt{1+(F/\overline{m_0}.c)^2\,.t^2}}.$$

Remark that we must consider only the positive signal, since $\overline{m_0} < 0$ and so [in (10)], both force F and time t will be positive, as presupposed. Therefore, we'll maintain for antiparticles the usual expression

$$u = c.\frac{\left(F/\overline{m_0}.c\right).t}{\sqrt{1+\left(F/\overline{m_0}.c\right)^2.t^2}}; \tag{11}$$

however, in this case, due to the *negative mass* of the antiparticle $\overline{\mathbf{P}}$, the velocity u will also be negative – but exactly symmetrical to the velocity of an homologous particle under equal circumstances. Symmetrically, too, remark that, according to this expression,

$$\lim_{t\to\infty} u = -c$$

Finally, we must emphasise an extraordinary property deduced from (10) or (11): **for antiparticles**, **force (like linear momentum) and induced velocity have opposite directions**. In other words, this means that *if we push an antiparticle forward, it will go backwards!*

3.2 Photons and antiphotons

The pair *photon - antiphoton* is a special case of pair *particle - antiparticle*. None of its elements has a proper mass or kinetic energy. However, according to the general rule, a **photon** in S" – that is to say, in S^* – will be detected in S as an **antiphoton** with negative energy and frequency. The development of pseudotachyonic theory shows that, in spite of what Quantum Electrodynamics sustain, **photon and antiphoton are not equivalent.** This means that the effect of these particles upon material ones, and vice-versa, *is not the same*. This is valid, for instance, in what concerns gravity or the Compton effect involving particles and antiparticles [see section 5].

Here, Relativity meets Quantum Mechanics; Pseudotachyonic Relativity shall do it quite well. Even a brief analysis of the present subject concerns the pseudotachyonic transformation of a wave which phase is $\varphi = k.x - \nu.t$. From the quantum relations

$$E = h.\nu \quad \text{and} \quad p = h.k = h/\lambda$$

– which we'll consider valid in all reference frames – together with equations (5), we obtain

$$\begin{cases} \nu^* = \frac{k_x.c-\nu.\beta}{\alpha} \\ k_x^* = \frac{k_x.\beta-\nu/c}{\alpha} \\ k_y^* = k_y \\ k_z^* = k_z. \end{cases} \tag{12}$$

Once again, the expressions are similar in the inverse transformation $S^* \to S$. If the propagation is parallel to the xx axis, we may just write

$$\begin{cases} \nu^* = \frac{k.c-\nu.\beta}{\alpha} \\ k^* = \frac{k.\beta-\nu/c}{\alpha}. \end{cases} \tag{12.a}$$

It's an easy step to conclude that, in pseudotachyonic transformations, the phase φ is anti-invariant ($\varphi = -\varphi^*$) and also that, if S' is the paraframe of S*,

$$\begin{cases} \nu^* = -\nu' \\ k^* = k'. \end{cases}$$

this means, since $u = \nu/k$ is the phase velocity of the wave, that $u^* = -u'$ (which, we have seen it in [1], is true for any velocity). Remark that the negative value for the transformed frequency ν^* is related to the pseudotachyonic inversion of time; it corresponds to a certain number of vibrations per -1 second.

Consider now a **photon** emitted by a **tachyonic source** (with velocity v). Using the equations (12) and (12.a), one can demonstrate that the expressions for the **longitudinal** and **transversal pseudotachyonic Doppler effects** are respectively

$$\nu = -\nu^*.\sqrt{\tfrac{c\mp\hat{v}}{c\pm\hat{v}}} \quad \text{and} \quad \nu = -\nu^*.\sqrt{1-(\hat{v}/c)^2}, \tag{13}$$

using the upper signs if the source is moving away of the observer, with a *detection velocity* $\hat{v} = c^2/v$; the lower ones if the source is coming closer. Both the results are symmetrical to those obtained for the bradyonic Doppler effect. They both mean that the photon is detected as an **antiphoton**. Since $E = h.\nu$, we may also conclude that, as any antiparticle, an antiphoton must have negative energy.

Remark that one cannot really distinguish *radiation* from *antiradiation* as far as *wavelength* is concerned because wavelength, supposed positive in S^*, will also be positive in our frame S. In one case or the other, the Doppler effects are expressed by

$$\lambda = \lambda^*.\sqrt{\tfrac{c\pm\hat{v}}{c\mp\hat{v}}} \quad \text{and} \quad \lambda = \tfrac{\lambda^*}{\sqrt{1-(\hat{v}/c)^2}}\,. \tag{13.a}$$

However – and this is very important – a beam of antiradiation, because of its negative energy, should be deviated by a *material* gravity field inversely to a beam of radiation.

4 Invariants, anti-invariants and electric charge

Since, in regard to time, a pseudotachyonic frame S^* and its paraframe *S'* behave symmetrically, we must admit that the element of *proper time* of a particle (moving in relation to S with velocity $\mathbf{u}$) is in pseudotachyonic transformations *anti-invariant*: $d\tau = -dt^*.\sqrt{1-(u^*/c)^2}$. Remark that this agrees with equation (6) in [1].

Now, the quadrivector (in tensorial notation) [2]

$$c^a = \rho^0.\tfrac{dx^a}{d\tau} \quad (a = 1, 2, 3, 4)$$

is the density of current charge, ρ^0 being the *proper electric charge density* in a certain point, measured by a local observer, and $\frac{dx^a}{d\tau}$ the quadrivector velocity (in relation to τ). So, for bradyonic transformations, in which $d\tau = dt.\sqrt{1-(u/c)^2}$,

$$c^a = \rho^0.\frac{dx^a}{dt}.\frac{dt}{d\tau} = \frac{\rho^0}{\sqrt{1-(u/c)^2}}.\frac{dx^a}{dt}.$$

Defining $\rho = \rho^0/\sqrt{1-(u/c)^2}$ as the *charge density* measured in the frame S, in which the charge has a velocity u at the instant t, we may write

$$c^a = \rho.\tfrac{dx^a}{dt} = \rho.u^a \quad \text{or} \quad \left\{\begin{array}{l} c^1 = \rho.u_x \\ c^2 = \rho.u_y \\ c^3 = \rho.u_z \\ c^4 = i.c.\rho. \end{array}\right.$$

The quadrivector c^a also transforms according to the bradyonic and pseudotachyonic laws for a generic contravariant quadrivector $\mathbf{A}^a$, (2) and (3.a). The result for the *time component* of c^a is

$$c^{*4} = \frac{i.c^1 - \beta.c^4}{\alpha} = i.c.\rho.\frac{u_x/c - \beta}{\alpha};$$

but $c^{*4} = i.c.\rho^*$, and, as a consequence,

$$\rho^* = \rho.\frac{u_x/c - \beta}{\alpha}. \tag{14}$$

It follows that, inversely to what occurs in bradyonic transformations, in pseudotachyonic transformations the proper charge density ρ^0 is *anti-invariant.* In fact, we may deduce from velocity transformation — equations (7) in [1] — the relation

$$\frac{\sqrt{1-(u/c)^2}}{\sqrt{1-(u^*/c)^2}} = \frac{\beta - u_x/c}{\alpha}, \tag{A}$$

which means, according to (14), that

$$\rho^*.\sqrt{1-(u^*/c)^2} = -\rho.\sqrt{1-(u/c)^2} = -\rho^0.$$

Now, due to the inversion of the time axis, the element of quadridimensional volume – which is invariant in bradyonic transformations – is also anti-invariant in the pseudotachyonic case ($dx^*.dy^*.dz^*.dt^* = -dx.dy.dz.dt$); but then, reminding that

$$\frac{dt^*}{dt} = \frac{u_x/c - \beta}{\alpha}$$

and applying the equality (A), the element of tridimensional volume, in ordinary space, transforms according to the equation

$$\frac{dv}{\sqrt{1-(u/c)^2}} = \frac{dv^*}{\sqrt{1-(u^*/c)^2}}. \tag{15}$$

On the other hand, since

$$\begin{cases} e = \rho.dv \\ e^* = \rho^*.dv^* \end{cases}$$

are the values obtained respectively in S and in S^* for the *electric charge* of a material particle, it results that

$$e = -e^*. \tag{16}$$

In short, *electric charge* (invariant in bradyonic transformations) is *anti-invariant* in pseudotachyonic transformations. So, **the electric charge of an antiparticle $\overline{\mathbf{P}}$ must be *opposite* to that of its homologous $\mathbf{P}$**.

Notice that, if we apply this result in a Newtonian approach, together with negative masses for antiparticles, for instance to the problem of deflection of a positron beam, it conduces to a wrong conclusion. As a matter of fact [5], if an electron beam enters perpendicularly in an uniform electric field $\mathbf{E}$, along the xx axis, with velocity v_0 then, despising gravitational force, its movement is described in function of time t by

$$x = v_0.t \quad \text{and} \quad y = \tfrac{1}{2}at^2 = \tfrac{eE}{2m}t^2;$$

so, eliminating the parameter t, we obtain for deflection y the equation

$$y = \frac{eE}{2mv_0^2}x^2.$$

Since the deduction of these equations *seems* exactly the same for a positron beam and considering that

$$\overline{e} = -e \quad \text{and} \quad \overline{m} = -m,$$

we would wrongly conclude that the deflection of both beams is the same. In truth, as experience shows, deflection is opposite for electron and positron beams. The problem lies in the premises of electrical fields characterisation because – in terms of Pseudotachyonic Relativity – we cannot directly apply Coulomb law,

$$F = \frac{1}{4\pi\epsilon_0}.\frac{q_1 q_2}{r^2},$$

to antiparticles, in the same way we may not apply Newton gravitational law. Both laws describe *observed effects* in *material* particles, not what's really going on (I mean the physical interaction mechanics). In both cases, the problem must be faced from another point of view, but this is the subject of a generalised and revised Field Theory I will not deal with in this paper. Anyway, one must be aware that the study of several issues requires new approaches.

Finally, since there is no real difference in the nature of homologous particles (as mentioned before, its "appearance" depends only on the reference frame) and since the pseudotachyonic transformation of the electric charge is anti-invariant, this may explain the **universality of charge quantization**: electron's and proton's charges must be equal in modulus because the *"element of charge"* is always the same. As a matter of fact, we may conceive that a proton 'contains' in itself a positron or, let's say, its 'essence'; this positron shows off in the transition *proton → neutron*: $\mathbf{p} \rightarrow \mathbf{n} + \bar{\mathbf{e}} + \nu_e$. By the way, remark that the neutron's inertial mass must be bigger than the proton's mass (as it is experimentally proved), since the positron mass is *negative* and the neutrino mass probably null.

I know that this statement contradicts the reasonable orthodox point of view, which (even before the *quark* theory) sustains that, comparing weak declines with meson disintegrations, *"we must not consider a neutron as a complex particle composed by a proton and an electron. We also must not consider that a proton is composed by a neutron and a positron. We don't deal with an ejection of ready-made particles but instead with the creation of new particles* (e^+, e^-, ν) *leading along with the transformation $n \rightleftharpoons p$ (in the same way that a light quantum emitted by an atom doesn't pre-exist but appears with the conversion of radiant energy)"* [7].

I propose a different explanation, briefly clarified here. If we may assume – for many reasons, for instance the universality of charge quantization or proton stability – that this one 'contains' a positron, we must not conclude that a neutron likely 'contains' an electron. In fact, this hypothesis conduces to an incoherent picture. If this was the case, the neutron should also 'contain' a positron, in order to annul the total charge. Besides the enormous instability this would produce (leading to the mutual annihilation of those components and so of the particle itself), it's incompatible with the proton decay: if the proton turns into a neutron by 'loosing' a positron, how could the neutron contain another one? The whole reasoning means that the inverse weak transitions

$$\mathbf{p} \rightarrow \mathbf{n} + \overline{\mathbf{e}} + \nu_e \quad \text{and} \quad \mathbf{n} \rightarrow \mathbf{p} + \mathbf{e} + \overline{\nu}_e$$

are indeed quite different phenomena. The first one consists on a disaggregation of an elementary particle (a proton), turning it into a neutron. The second one seems to imply a spontaneous *double pair creation*, the created electron and antineutrino being ejected and the positron and neutrino, remaining in the particle, creating a new one: a proton (and this in obedience to the *creation principle* presented ahead – subsection 6.2).

5 The Compton effect

We'll study now the generalization of the Compton effect — the scattering of photons by free electrons — to the case of antiparticles. The Compton effect consists on the collision of a non-massive particle ($\mathbf{P_1}$) with a massive particle ($\mathbf{P_2}$), for instance an electron, supposed immobile in the reference frame S. The result of the collision are two particles ($\mathbf{P_3}$ and $\mathbf{P_4}$), respectively identical to $\mathbf{P_1}$ and $\mathbf{P_2}$ but which states of movement are different from the start.

It's convenient to express these movements through each respective linear momentum. In the subsequent analysis, it will be enough to consider that antiparticles have negative masses and concomitantly that (see section 3.1) their linear momentum vector $\mathbf{p}$ and their velocity vector $\mathbf{u}$ have opposite orientations. For practical use, then, in the following formulae we'll make ($i = 1, 2$)

$$\begin{cases} k_i = 0 & \text{for particles; and} \\ k_i = 1 & \text{for antiparticles.} \end{cases}$$

Let φ and θ be the angles the vectors $\mathbf{p_3}$ and $\mathbf{p_4}$ form with the vector $\mathbf{p_1}$ [see figure 2]. If the incoming particle is a *photon* (positive energy), the **scattering angles** (or displacement angles) for both particles – relatively to the xx axis positive sense – are

$$\begin{cases} \varphi_s = \varphi & \text{for the photon; and} \\ \theta_s = k_2.180^\circ + \theta & \text{for the electron/positron.} \end{cases}$$

If the incoming particle is a *antiphoton* (negative energy), the scattering angles for both particles – once more relatively to the xx axis positive sense [see figure 3] – are

$$\begin{cases} \varphi_s = -\varphi & \text{for the antiphoton; and} \\ \theta_s = (1 - k_2)\,.180^\circ - \theta & \text{for the electron/positron.} \end{cases}$$

Finally, we may unite these two equation system in a single one:

$$\begin{cases} \varphi_s = (-1)^{k_1}.\varphi \\ \theta_s = k_1.180^\circ + (-1)^{k_1}(k_2.180^\circ + \theta) \end{cases} \tag{17}$$

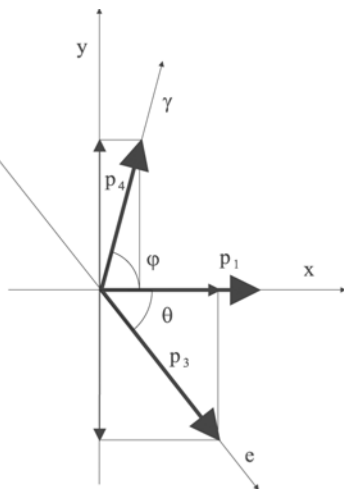

Figure 2: Momentum vectors for a photon colliding with an electron or a positron

Now, in obedience to the conservation of the linear momentum, it must be [4]

$$\mathbf{p_4} = \mathbf{p_1} - \mathbf{p_3}$$

and so (considering $E_1 = E$ and $E_3 = E'$ for the non-massive particle and also $E_2 = E_0$ for the massive one)

$$p_4^2 = p_1^2 + p_3^2 - 2\mathbf{p_1}.\mathbf{p_3} = \frac{1}{c^2}.\left(E^2 + E'^2 - 2E.E'\cos\varphi\right); \qquad \text{(a)}$$

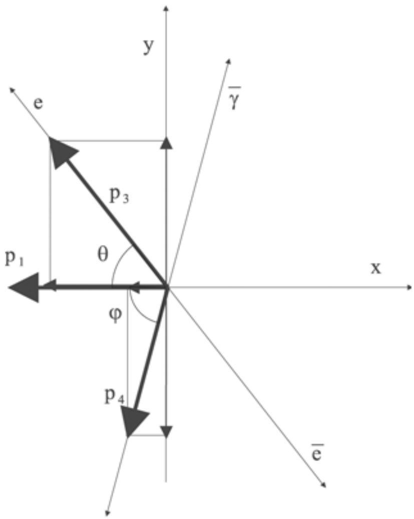

Figure 3: Momentum vectors for an antiphoton colliding with an electron or a positron

on the other hand, the conservation of energy law implies that

$$E + E_0 = E' \pm \sqrt{E_0^2 + c^2.p_4^2},$$

which means

$$p_4^2 = \frac{1}{c^2}.\left[E^2 + E'^2 - 2E.E' + 2\left(E - E'\right).E_0\right]; \qquad \text{(b)}$$

finally, equalizing the second terms of equations (a) and (b), it results

$$\left(E - E'\right).E_0 = E.E'\left(1 - \cos\varphi\right),$$

or

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{E_0}\left(1 - \cos\varphi\right), \qquad (18)$$

which is the well known mathematical expression for the Compton effect. Notice that *this expression must also be valid for antiparticles* because the signs of the variables energy or linear momentum have no relevance in its deduction. Since $E = \frac{h.c}{\lambda}$ and $E' = \frac{h.c}{\lambda'}$, we may write (for $\Delta\lambda = \lambda' - \lambda$)

$$\Delta\lambda = \frac{h}{m_0.c}\left(1 - \cos\varphi\right), \tag{18.a}$$

in the condition that we consider for antiphotons a ***negative*** *wavelength* (this means, contrary to its propagation velocity). Anyway, as we have seen before, in the calculus we'll use the angle

$$\varphi = (-1)^{k_1}.\varphi_s$$

corresponding to a certain scattering angle φ_s (considering $k_1 = 0$ for *photons* and $k_1 = 1$ for *antiphotons*).

Other interesting equations are those obtained for the kinetic energy E_k, the angle θ (for linear momentum) and the factor $\beta = v/c$ of the hit particle ($\mathbf{P_4}$):

$$E_k = E - E' \qquad \Rightarrow \quad E_k = \tfrac{\Delta\lambda}{\lambda.\lambda'}.hc\,;$$

$$\beta^2 = 1 - \tfrac{E_0^2}{(E_0+E_k)^2}, \quad \text{or} \quad \beta^2 = \tfrac{E_4 - E_0^2}{E_4^2};$$

$$\tan\theta = \tfrac{\lambda.\sin\varphi}{\lambda.\cos\varphi - \lambda'}.$$

The angle θ may also be obtained using Debye's formula [6], adapted with the - sign to the present analysis:

$$\cot\theta = -\left(1 + \frac{h}{m_0 c\lambda}\right)\tan\frac{\varphi}{2}$$

As it is known, this equation clearly shows that, for an incident photon, which may be scattered with any angle ($-\pi \leq \varphi_s \leq \pi$), an electron is confined within the space frontal region ($-\pi/2 \leq \theta \leq \pi/2$). This means that the electron always moves forwards. But remark that, for an incident **antiphoton**, since, for the **electron**, $\theta_s = 180° - \theta$, the hit particle moves with a scattering angle within the interval ($\pi/2 \leq \theta \leq 3/2\pi$), thus confined to the posterior region; or, in other terms, **it moves backwards!** Another important feature is that, in this case, **the wavelength diminish**, that is to say **the**

energy of the antiphoton increases (both in modulus). This negative energy increment counterbalance exactly the kinetic energy of the electron set in motion. The results are the same if we consider respectively an incident antiphoton or a photon against a positron (the photon's energy also increases).

6 The pair annihilation/creation problem

6.1 The collision of homologous particles

According to Dirac's theory, the collision of an electron with an antielectron must give rise to their mutual annihilation, energy springing on the form of two light quanta. The theory also predicted the inverse process [9][6]. Let us see now how we may come to similar conclusions, despite some serious theoretical divergences.

One must recall this remarkable property of antiparticles (see section 3.1): linear momentum vector **p** and either velocity vector **u** either force vector **F** have opposite directions; so, *"if we push an antiparticle forward, it will go backwards".* Consider now a first particle, an electron **e**, immobile in 'our' frame *(S)* and a second one, an homologous $\overline{\mathbf{e}}$, moving towards it with velocity $u_2 = u$.

Figure 4: Colliding homologous particles in the rest frame of the 'particle'

Colliding with **e**, the positron $\overline{\mathbf{e}}$ will partially transmit to it its linear momentum; however, since this linear momentum is opposite to the movement of $\overline{\mathbf{e}}$, the electron should not withdraw from the positron but instead accelerate over it. This one would then react *negatively* to the pressure of **e**, also hastening upon it. This means that the two particles would tend to 'fuse' instantly one with the other – if things would happen this way. But they don't. Because, for symmetry reasons and contrarily to what may occur if the two particles are not homologous ones, the product of such 'fusion' couldn't be neither a particle nor an antiparticle; it couldn't then have a definite displacement direction, clearly related to the total linear momentum, which should remain not null. That's maybe why the 'overlapping' must occur at the cost of the transmutation of both particles into 'packets of pure energy', which can perform such phenomenon.

This conclusion remains obviously the same whatever the relative state of movement of the colliding particles is. Contrarily to the usual prediction, however, instead of a pair of photons, the above mentioned packets must be *a photon and an antiphoton* – that is to say, a *mixed energy.*

We'll study now this subject from a quantitative point of view. In the particle frame, S, we'll have

$$\text{1)particle}\mathbf{Q_1}\ (\mathbf{e}) \quad \begin{cases} E_1 = E_0 \\ p_1 = 0; \end{cases} \tag{a}$$

$$\text{2) particle } \mathbf{Q_2}\ (\overline{\mathbf{e}}) \quad \begin{cases} E_2 = -\frac{\beta}{\alpha}.E_0 \\ p_2 = -\frac{E_0}{\alpha.c} = p; \end{cases} \tag{b}$$

considering

$$\beta = 1/\beta_2 = c/u$$

(which means that $v = \hat{u}$ is the velocity of the tachyonic frame associated to S^*, the positron $\overline{\mathbf{e}}$ proper frame).

In the pseudotachyonic frame S^*, $\overline{\mathbf{Q}}_2 = \mathbf{e}$ is a particle (identical to the electron $\mathbf{e}$ in S) and $\overline{\mathbf{Q}}_1 = \overline{\mathbf{e}}$ is its homologous antiparticle. According to equations (5.a), it will be

$$1')\text{ particle } \overline{\mathbf{Q}}_\mathbf{1}\ (\overline{\mathbf{e}}) \quad \begin{cases} E_1^* = \frac{0-\beta}{\alpha}.E_1 = -\frac{\beta}{\alpha}.E_0 = E_2 \\ p_1^* = \frac{\beta.0-E_1/c}{\alpha} = -\frac{E_0}{\alpha.c} = p_2; \end{cases} \tag{c}$$

$$2')\text{ particle } \overline{\mathbf{Q}}_\mathbf{2}\ (\mathbf{e}) \quad \begin{cases} E_2^* = \frac{1/\beta-\beta}{\alpha}.E_2 = -\frac{\alpha}{\beta}.E_2 = E_0 = E_1 \\ p_2^* = \frac{\beta-\beta}{\alpha}.p_2 = 0 = p_1; \end{cases} \tag{d}$$

and, concerning velocities,

$$\begin{cases} u_1^* = u_2 \\ u_2^* = u_1. \end{cases}$$

These results mean that, if we consider *time flowing negatively in* S^*, there is, from frame S to S^*, an absolutely symmetrical inversion of roles; that is to say, the situation is exactly the same in both frames. That's why if a *photon* is created in S, an *antiphoton* shall appear in S^*; but then, symmetrically, a *photon* must be created in S^*, and so an *antiphoton* in *S!*

Figure 5: Colliding homologous particles in the pseudotachyonic rest frame of the 'antiparticle'

Now, it results from the anterior equations, (a) to (d), that

$$\begin{cases} E_1^* + E_2^* = E_1 + E_2 = E \\ p_1^* + p_2^* = p_1 + p_2 = p, \end{cases}$$

E and $\mathbf{p}$ being respectively the system $\mathbf{P} + \overline{\mathbf{P}}$ total energy and total linear impulse, in both frames S and S^*:

$$\begin{cases} E = (1 - \frac{\beta}{\alpha}).E_0 \\ p = -\frac{E_0}{\alpha.c}. \end{cases} \tag{e}$$

As a consequence, if $\mathbf{Q}_3$ and $\mathbf{Q}_4$ are the pair photon/antiphoton resulting from the annihilation, it must be

$$\begin{cases} E_3 + E_4 = E \\ E_3^* + E_4^* = E, \end{cases} \tag{f}$$

and also

$$\begin{cases} p_3 + p_4 = p \\ p_3^* + p_4^* = p, \end{cases} \tag{g}$$

According to the formula for energy transformation in (5.a),

$$\begin{cases} E_3^* = \frac{\beta_3 - \beta}{\alpha}.E_3 \\ E_4^* = \frac{\beta_4 - \beta}{\alpha}.E_4, \end{cases}$$

applying these equations in (f), we obtain the system

$$\begin{cases} E_3 + E_4 = E \\ \frac{\beta_3-\beta}{\alpha}.E_3 + \frac{\beta_4-\beta}{\alpha}.E_4 = E, \end{cases} \tag{f$'$}$$

which solution is

$$\begin{cases} E_3 = \frac{\beta_4-(\alpha+\beta)}{\beta_4-\beta_3}.E \\ E_4 = \frac{\beta_3-(\alpha+\beta)}{\beta_3-\beta_4}.E, \end{cases} \tag{19}$$

or, in view of (e),

$$\begin{cases} E_3 = \frac{\beta_4.(\alpha-\beta)+1}{\alpha.(\beta_4-\beta_3)}.E_0 \\ E_4 = \frac{\beta_3.(\alpha-\beta)+1}{\alpha.(\beta_3-\beta_4)}.E_0. \end{cases} \tag{18.a}$$

In its turn, from the system

$$\begin{cases} p_3 + p_4 = p \\ \frac{\beta.\beta_3-1}{\alpha.\beta_3}.p_3 + \frac{\beta.\beta_4-1}{\alpha.\beta_4}.p_4 = p, \end{cases} \tag{g$'$}$$

we obtain

$$\begin{cases} p_3 = \frac{\beta_4.(\beta-\alpha)-1}{\beta_4-\beta_3}.p \\ p_4 = \frac{\beta_3.(\beta-\alpha)-1}{\beta_3-\beta_4}.p. \end{cases} \tag{20}$$

Now, since the pseudotachyonic transformation of $\pm c$ is $\mp c$, we conclude that *the photon and the antiphoton must have opposite velocities*; we'll considerer, then,

$$\begin{cases} \beta_3 = -1 \\ \beta_4 = 1, \end{cases} \quad \text{and symmetrically} \quad \begin{cases} \beta_3^* = 1 \\ \beta_4^* = -1; \end{cases}$$

in these terms, we may finnaly write

$$\begin{cases} E_3 = \frac{(\alpha-\beta)+1}{2\alpha}.E_0 \\ E_4 = \frac{(\alpha-\beta)-1}{2\alpha}.E_0. \end{cases} \tag{18.b}$$

One verifies that $E_3 > 0$ and $E_4 < 0$; so, $\mathbf{Q}_4$ is the antiphoton and it moves in the same direction then the incoming positron $\bar{\mathbf{e}}$.

It is possible to evaluate these results, in a symmetrical manner for the photon and the antiphoton, considering – for each value of β_2 – the bradyonic reference frame *S'* in which the positron and the electron have equal but opposite velocities. The velocity of this frame is given by $\beta_{S'} = \frac{1-\sqrt{1-\beta_2^2}}{\beta_2}$. We conclude that, in this new frame, **the total energy is always null, but not the total linear momentum**; that's why the pair electron/antielectron doesn't simply disappear without any traces.

One must remark that, in the qualitative and quantitative study of the collision positron/electron, we considered *time flowing negatively in S**. But, although the deductions above-presented are independent from the temporal direction we confer them, we must now take this direction into account. As we have seen in section 1, a positive time flow in the reference frame S corresponds to a negative flow in S^* and vice-versa. So, as in the 'light beam experience' [1], the event 'annihilation' in S appears in S^* as its inverse process: *the* ***creation*** *of a pair electron/positron trough the energy of a colliding pair photon/antiphoton.* In virtue of the principle of relativity, this process – the creation of a pair particle/antiparticle – must happen in any reference frame, obeying to similar equations. We see, then, that annihilation and creation are imperatively related phenomena.

This agrees with the canonical Quantum Theory. But serious divergence towards it are evident. For instance, Quantum Theory predicts that, the energy of both the electron and the positron being supposed positive, the total energy cannot be null in any reference frame. As a result, the energy liberation in the *under limit* (the velocities of the particles being negligible) is $2E_0$, equally divided by the two photons produced. However, this is obviously in contradiction with (18.b), since, according to those equations,

$$\begin{cases} \lim\limits_{u\to 0} E_3 = 0 \\ \lim\limits_{u\to 0} E_4 = 0, \end{cases}$$

which means that in the *under limit* there wouldn't exist any liberation of energy at all! This conclusion – quite troublesome, I confess, because it *seems* to be also contrary to experimental evidence – imposes itself, nevertheless, for its physical and mathematical logic in the context of the present theory. As a matter of fact, since the velocities of the pair photon/antiphoton are opposite, the linear momentum of both particles must have the same direction (opposite to the movement of the positron). But $p_3 + p_4 = p$, and so the unique solution for the equation $p_3 + p_4 = 0$, corresponding to the above mentioned *under limit*, consists on the *inexistence* of the pair photon/antiphoton: $p_3 = p_4 = 0$. Anyway, under these circumstances, the particles don't even collide!...

This *under limit* relates to the threshold energy involved in the inverse process, the **creation** of a pair electron/positron *through a colliding pair photon/antiphoton*. But, in fact, an amazing conclusion arises: in the creation of a pair electron/positron, high energies (in modulus) for the pair photon/antiphoton are not really required! On the contrary, in terms of E_0, they may be very low! For instance, with a pair photon/antiphoton with energy $E = \pm 0,1015\, E_0$ it should be possible to create a pair of homologous particles with proper energy $\pm E_0$ and opposite velocities equal to $20\%\, c$. On the other hand — should we say, in compensation — the required energy must be *mixed*. We'll return to this point in subsection 6.3, paragraph B.

6.2 Two homologous particles system

One must be aware that, according to this theory, a simple system formed by two homologous particles behaves physically (and mathematically) in a quite peculiar way, differently from what one should expect or is used to. Therefore, usual reasoning – concerning systems formed by two or more particles with positive energy – may fail and conduce to wrong conclusions.

Let's examine, for instance, the concept of ***C frame*** [4]. If we considerer a system of n particles, in ordinary Relativity mechanics the *C reference frame* is defined as the one in which the total linear momentum is null; its velocity, in a certain frame S, is the system's velocity, given by

$$v_C = \frac{c^2.\mathbf{p}}{E} = \frac{\mathbf{p}}{M},$$

$\mathbf{p} = \sum_i \mathbf{p}_i$ and $E = \sum_i E_i = \sum_i m_i.c^2 = Mc^2$ being respectively the total linear momentum and the total energy of the particles system in the frame S. We assume that, in this concept, total and partial energies and masses are positive.

This concept of *C frame* is fundamental in the standard approach to the problem of *pair creation* examined in the next subsection: the creation of two homologous particles by means of two colliding particles of the same kind. The threshold energy for this process to take place is logically defined considering that the four resulting particles are all at rest in their *C frame*. We'll see now that, alas!, we cannot apply this concept, and so this point of view, in a pseudotachyonic approach of the problem (if we do it, besides contradictions, it conduces to a false equation and a false solution). As a matter of fact, we cannot define a *C frame* if the system is – for simplicity – composed by a pair of homologous particles. Let's see why.

Consider two homologous particles, $\mathbf{P}$ and $\overline{\mathbf{P}}$, moving with the same velocity $\mathbf{u}$ [see figure 6]. It results

$$\begin{cases} \mathbf{p} = m.\mathbf{u} \\ \overline{\mathbf{p}} = \overline{m}.\mathbf{u} = -m.\mathbf{u} = -\mathbf{p}; \end{cases}$$

as a consequence, the total linear momentum is null and so is the total energy, $E + \overline{E}$.

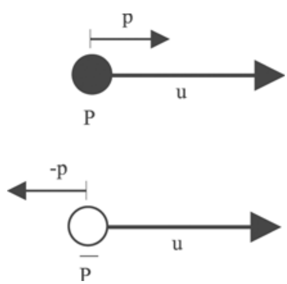

Figure 6: Two homologous particles moving with the same velocity

In another bradyonic frame *S'*, which velocity is $\mathbf{v}$ – for simplicity, supposed parallel to $\mathbf{u}$ – it results

$$u' = \frac{u - v}{1 - u.v/c^2}$$

for both particles. It's easy to demonstrate that

$$\begin{cases} \overline{\mathbf{p}}\,' = -\mathbf{p}' \\ \overline{E}\,' = -E' \end{cases}$$

this being true even if $\mathbf{v}$ isn't parallel to $\mathbf{u}$. We obtain the same coherent conclusion in a pseudotachyonic transformation, according to formula (5.a):

$$\begin{cases} \overline{\mathbf{p}}\,^* = -\mathbf{p}^* \\ \overline{E}\,^* = -E^* \end{cases}$$

So, in one case or the other, the total linear momentum remain null and, of course, the total energy too; but, contrary to the vector variable $\mathbf{p}$, $\overline{E} = -E$ also implies that the velocities $\mathbf{u}$ and $\overline{\mathbf{u}}$ are equal in modulus but not necessarily in direction. This means that the total energy may be null in one reference frame but not in another one (as we saw, for instance in the last subsection). So, we may conclude that:

if the total linear momentum for a two homologous particles system is null in a reference frame, it is null in any other frame.

Therefore, in these conditions, the concept of *C frame* has no great sense (unless we identify it with the frame in which the two particles are at rest).

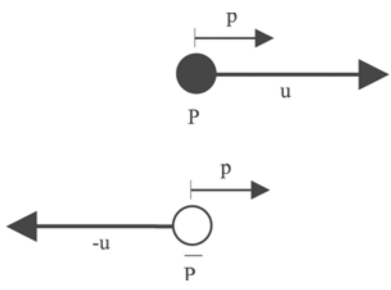

Figure 7: Two homologous particles moving with opposite velocities

Suppose now that the two homologous particles move with opposite velocities [see figure 7]: $\overline{\mathbf{u}} = -\mathbf{u}$ and $\overline{\mathbf{p}} = \mathbf{p}$. Since $\overline{E} = -E$ the total energy is, once again, null in S but not the total linear momentum, which is equal to $2\mathbf{p}$. In the frame *S'* the transformed velocities are

$$u' = \frac{u-v}{1-u.v/c^2} \quad \text{and} \quad \overline{u}' = \frac{\overline{u}-v}{1-\overline{u}.v/c^2} = -\frac{u+v}{1+u.v/c^2}$$

and it will be

$$\begin{cases} p' = \frac{p-v.E/c^2}{\sqrt{1-\beta^2}} \\ E' = \frac{E-v.p}{\sqrt{1-\beta^2}} \end{cases} \quad \text{and} \quad \begin{cases} \overline{p}' = \frac{\overline{p}-v.\overline{E}/c^2}{\sqrt{1-\beta^2}} = \frac{p+v.E/c^2}{\sqrt{1-\beta^2}} \\ \overline{E}' = \frac{\overline{E}-v.\overline{p}}{\sqrt{1-\beta^2}} = -\frac{E+v.p}{\sqrt{1-\beta^2}}. \end{cases}$$

One can verify that, as it happens in general [see equation (6)], $E' = p'.c^2/u'$ and $\overline{E}' = \overline{p}'.c^2/\overline{u}'$. The total linear momentum and energy will be given by

$$\begin{cases} p' + \overline{p}' = \frac{2p}{\sqrt{1-\beta^2}} \\ E' + \overline{E}' = -\frac{2v.p}{\sqrt{1-\beta^2}} \end{cases}$$

Remark that the total energy results *negative* if v and p have the same direction (both positive or both negative) – which means that in the frame *S'* the antiparticle moves faster than the particle. On the contrary, if v and p have opposite directions, in *S'* the particle moves faster than the antiparticle and the total energy is positive.

Now, the expression for the total linear momentum implies that it may only be null if $p = 0$ – that is to say, if the pair of homologous particles is at rest in the frame S, in which case we return to the precedent situation. If not, we conclude that:

> **if, in a certain reference frame, two homologous particles move with equal but opposite velocities, there isn't any frame in which the total linear momentum is null.**

This conclusion may be extended to **any state of movement** for the particles – except in case both elements have the same (parallel) velocity, which is coherent with the first quoted statement –, since we'll may always find one frame in which either the particle or the antiparticle is at rest. And then we come to the last mentioned frame *S'*, with either velocity $\mathbf{v} = \mathbf{u}$ or $\mathbf{v} = \overline{\mathbf{u}} = -\mathbf{u}$.

Finally, remark that, under these general conditions, **there isn't any *C frame!***

All this reasoning concerning a pair of homologous particles, together with the conclusions presented in the previous subsection, lead to the following **pair creation principle** of enormous importance:

> **In the creation of a pair of homologous particles – as well as in the inverse process, their annihilation – the total linear momentum is never null.**

It may happen that both energies cancel, the total energy for the system being null – we may indeed find a reference frame in which this happens (as referred in the previous subsection) – but never the linear impulse. Should this occur and the two particles simply wouldn't collide and annihilate each other – nor the inverse process (seen by a pseudotachyonic frame) would take place. The fundamental consequence is that:

> **A pair of homologous particles – or, in general, "mixed energy" – cannot be created from nothing; inversely, a colliding pair will never turn into nothing.**

6.3 Pair creation trough the collision of two identical particles

A. The standard approach

The standard approach to the *"threshold energy"* involved in the pair creation trough the collision of two identical particles (for instance protons) is based on the assumption that the state of movement after the collision is the same for all the four particles [4]. Moreover, the "threshold energy" – the kinetic energy necessary for an incident proton to obtain the desired result – is the one required so the four resulting particles are at rest in the *C frame*, which is moving with the velocity of this system of particles. In the laboratory frame *(L)*, the four particles move with identical speed, energy and linear momentum.

If one of the initial protons is at rest at the laboratory frame L (with energy E_0) and the incident proton is moving with a linear momentum equal to **p**, the principle of total energy conservation equalize the total energy before and after the collision:

$$\sqrt{E_0^2 + p^2c^2} + E_0 = 4.\sqrt{E_0^2 + (p/4)^2\, c^2}.$$

Raising to the square both members of this equation, two times, we obtain its solution: $p = 4\sqrt{3}E_0/c$. The incident proton's total energy is $E = 7E_0$; this means that its kinetic energy – the "threshold energy" we're looking for – is $E_k = 6E_0$ and its velocity $v = 4\sqrt{3}/7.c = 0,9897.c$.

Sure, from a mathematical point of view, these are very pretty results. However some "dark clouds" may appear in the horizon if we meditate on their premises.

First of all, it seems quite odd that (in the *C frame*) two colliding protons that were moving at very high speed – equal but opposite – turn into four **immobile** particles! But even if we accept this, a serious physical problem arises: if the created pair is in touch why doesn't it immediately **annihilate** itself? Of course, one may argue that this is only an artificial threshold condition; but, if the kinetic energy of the incoming proton is higher than the threshold energy, the resulting four particles cannot obviously be **all** at rest in *C frame*. In this frame the two initial protons are – as before – exactly in the same circumstances; so, they must react identically: they may ricochet (in general, move symmetrically) or stay still. Anyway, if the created particles stay still, once again they should annihilate themselves; if not, to avoid this, the antiproton must drift away from the three protons. But in what direction? Why should it go one way instead of another? From a classical point of view, there seems to be no motif for this 'choice'! So, does the antiproton always stand still after the collision, the three protons moving away from it? Is this acceptable?

In brief, this reasoning seems to evidence the limits of classical determinism. But there are other major objections. For instance, **why** do energy (equal or superior to the "threshold energy") create a pair $\mathbf{p} + \overline{\mathbf{p}}$ instead of simply implicate a change in the state of movement of the former two particles? And why doesn't it create two pairs $\mathbf{p}+\overline{\mathbf{p}}$ or, for a lower threshold, a pair $\mathbf{e}+\overline{\mathbf{e}}$? It is experimentally proved that two protons colliding with very high energy may give rise to a *multitude* and also a *multiplicity* of other particles. But why is a particle **A** created, or a particle **B** and not another one (in satisfying conditions, of course)? If it's just a question of disposable (positive) energy, why doesn't free energy constantly create pairs of homologous particles in Nature?

One may also note that, considering the same threshold energy for a *colliding pair of antiprotons*, we should obtain the same resulting pair $\mathbf{p} + \overline{\mathbf{p}}$:

$$\overline{\mathbf{p}} + \overline{\mathbf{p}} \rightarrow \overline{\mathbf{p}} + \overline{\mathbf{p}} + \mathbf{p} + \overline{\mathbf{p}} \, ;$$

isn't this quite strange?

B. The pseudotachyonic approach ***(an open problem)***

The standard premises to the pair creation problem, in the search for the threshold energy, don't apply in a pseudotachyonic approach. First of all, in the *C frame* of the colliding protons (particles **1** and **2**), the total impulse is null and its total energy is

$$E_t = 2.\sqrt{E_0^2 + p^2.c^2} = 2E \; ,$$

$\pm p$ being the linear impulse for each colliding particle and E the corresponding energy. If, after the collision, all the four particles were at rest, the energy of the created pair would be null ($E_0 - E_0 = 0$) and so the total energy would result equal to $2E_0$ – thus violating the principle of energy conservation. Coherently this creation of a pair which total linear impulse is null would also violate the *pair creation principle* enounced in subsection 6.2. Besides, as we have seen, the concept of *C frame* doesn't apply to the pair $\mathbf{p} + \overline{\mathbf{p}}$ (after the collision) and so the simple application of the standard premises is here condemned to failure.

We must therefore come back in our reasoning. If we write

$$E_t = 2(E_0 + E_k) \; ,$$

then, after the collision and the creation of a pair (particles **3** and **4**, proton and antiproton respectively) – considering that, for symmetry reasons, $p'_1 = -p'_2$ and $E'_{k1} = E'_{k2} = E'_k$ – it should be

$$E_3 + E_4 + 2(E_0 + E'_k) = 2(E_0 + E_k) \; ,$$

or

$$E_3 + E_4 = 2(E_k - E'_k) \ .$$

This means that the energy responsible for the pair creation is the difference between the total kinetic energies of the colliding particles (before and after the collision). If $E_k = E'_k$ (an elastic collision, the two colliding protons ricocheting), then $E_3 = -E_4$ and also $p_3 = -p_4$; no pair would be created because this would violate the *pair creation principle*.

If, as in the standard approach, we consider for the threshold energy $p'_1 = p'_2 = 0$ (that is to say, $E'_k = 0$), then $E_3 + E_4 = 2E_k$; this is a **positive energy** and this fact introduces an **asymmetry** in the physical process; it would be **negative** if the colliding particles were antiprotons. If we now assume that this implicates a greater energy (in modulus) for the created proton – this is, $E_3 > |E_4|$, since $E_3 + E_4 = 2E_k$ or $E_3 = 2E_k + |E_4|$ –, then the total linear impulse cannot be null and this violates its basic conservation principle. This means that there must be *anything else* that counterbalance the linear impulse of **3+4** couple: at least, a *fifth particle*, probably a *photon* (anyway, a non-charged particle), this one being the physical manifestation of the mentioned asymmetry, according to the basic equation system:

$$\begin{cases} E_3 + E_4 + E_5 = 2E_k \\ p_3 + p_4 + p_5 = 0. \end{cases}$$

According to this hypothesis, the pair creation should consist in the following process:

$$\mathbf{p} + \mathbf{p} \rightarrow \mathbf{p} + \mathbf{p} + \mathbf{p} + \overline{\mathbf{p}} + \gamma \ ;$$

in the case of a pair creation through the collision of two *antiprotons*, the fifth particle should be the homologous of the precedent:

$$\overline{\mathbf{p}} + \overline{\mathbf{p}} \rightarrow \overline{\mathbf{p}} + \overline{\mathbf{p}} + \mathbf{p} + \overline{\mathbf{p}} + \overline{\gamma} \ .$$

Remark that, in truth, this fifth particle may change our picture: because of its energy contribution, it seems no longer necessary that

$E_3 > |E_4|$. Therefore, if we assume that – according to the *indiscernibleness of identical particles* established by Quantum Theory – we wouldn't be able to distinguish the created proton from the other two, it results that $p_3 = 0$, $E_3 = E_0$ and

$$\begin{cases} E_4 + E_5 = 2E_k - E_0 \\ p_4 + p_5 = 0. \end{cases}$$

We may then write, since $2E_k = 2(E - E_0) = 2.\left(\sqrt{E_0^2 + p^2.c^2} - E_0\right)$,

$$|p_4.c| - \sqrt{E_0^2 + p_4^2.c^2} = 2.\sqrt{E_0^2 + p^2.c^2} - 3E_0.$$

One may notice that $|p_4.c| - \sqrt{E_0^2 + p_4^2.c^2} < 0$ and so it's easy to conclude (applying this inequality to the second member of equation) that, *considering our threshold conditions,* it should be $|u| < \frac{\sqrt{5}}{3}c$ for the velocity of the two collinding protons, in their C frame; in the L frame of one of them, this corresponds to an incident velocity $|u'| < \frac{3}{7}\sqrt{5}c = 0,95832.c$, which is under the standard value $4\sqrt{3}/7.c = 0,9897.c$.

Is this true? Even if it is, the equation above has two independent variables (p and p_4). There's still *something missing* and it probably relates to the questions raised in the last paragraph of the standard approach analysis, mainly this one: why do kinetic energy turns into a pair of homologous particles?

I haven't yet been able to solve the problem. Maybe it cannot be resolved from a straight mechanical point of view. I conjecture that, **to create a pair of homologous particles, mixed energy is required** – and this is already suggested at the end of subsection 6.1. I have the feeling that the "threshold energy" required in the creation of a pair $\mathbf{p} + \overline{\mathbf{p}}$ through the collision of two protons relates to the *structure* of these particles; I mean, maybe it is the lower energy necessary to launch a process that begins with a forced transition

$$\mathbf{p} \rightarrow \mathbf{n} + \overline{\mathbf{e}} + \nu.$$

Negative energy appears in the form of a positron. If this antiparticle and the neutrino "loose" some of their very high energy before falling into the neutron and being "reabsorbed" by it, turning it again into a proton, these negative and positive energies may well create a pair proton + antiproton (since, we have seen it in subsection 6.1, the mixed energy may be very low compared to the rest energy of the created homologous particles).

Maybe this is just a fantasy or purely guess-work!... Anyway, either from a physical or a mathematical point of view, the resolution of the pair creation problem is apparently not so simple than the standard one. On the other hand, it may provide a more profound comprehension of this phenomena and others correlated to it. This is yet an open problem.

7 Time inversion and causality

Problems arisen by the possibility of time inversion are not simple ones. They have no easy answers. However, this doesn't mean we should take an expeditious way, denying the possibility itself by a sort of arbitrary postulate. In fact, like in other profound conceptual changes, we must revolutionize our own way of thinking and understanding the world. Richard Feynman, for instance, had no problem considering an *antiparticle* as a *"particle that goes back in time"*. This point of view perfectly agrees with the present theory, which enunciate it as a logical and consistent conclusion. Feynman applied this reversibility of time to the interpretation of some inverse phenomena, such as the annihilation/creation of a pair particle-antiparticle. The general procedure has a very strong consistency in Pseudotachyonic Relativity – in fact it may be a powerful instrument for reasoning – since it directly arises from the relationship between paraframes or between mutually pseudotachyonic frames. Important consequences may also be obtained from the transformation $S^* \leftrightarrow S$.

We have studied such a consequence in the problem of the light beam in [1], section 1.3. In that 'ideal experiment' time inversion doesn't bring a major problem because there is an obvious symmetry of the situation in relation to time flow. But, the fact is that this remark is valid for almost all the problems we deal with either in Newtonian or Relativity Physics: in both cases, the majority of equations are perfectly valid in a negative time flow. This means both the mechanics are, in essence, time-reversible. Anyway, this reversibility also implicates that often a *cause-effect* relationship, at a fundamental level, is not really very clear.

There are, however, some exceptions. Take, for instance, the above mentioned experience of the light beam. Now, instead of being reflected in b), imagine that the beam is *polarized*; this polarized beam goes its way and reaches a detector in a point (**C**) distant $2r$ from point **A**. We'll have, then:

c) Reception (in **C**):

$$\begin{cases} x_3^* = 2r^* \\ t_3^* = \frac{2r^*}{c} \end{cases} \Rightarrow \begin{cases} x_3 = 2r^*.\sqrt{\frac{\beta-1}{\beta+1}} = 2x_2 \\ t_3 = -\frac{2r^*}{c}.\sqrt{\frac{\beta-1}{\beta+1}} = 2t_2 \end{cases} \Rightarrow \; x_3 = -c.t_3.$$

Again, this result has an obvious interpretation, taking into account the *time inversion* in S. Yet, because of this inversion, the experience would appear to us in a strange way: we would see a polarized (anti)radiation going through a polarizer and get out as an unpolarized one! This is amazing but must be truth. Together with the predictable negative deviation by a material gravity field, this is one way to distinguish antiradiation from radiation. Inverse behaviours, quite bizarre like this one, certainly characterizes negative energy – antiradiation or antimatter –, as we'll see right ahead.

Nevertheless, the truth is that a cause-effect relationship and irreversibility itself get a major concrete sense in the statistic mechanics of complex systems, mainly in the concept of entropy and in the second law of Thermodynamics. Once again, we may analyse this problem resorting to a frame S – "our own" *material* reference frame – and its paraframe, *antimaterial*, S^*, which velocity is null. As we have seen in subsection 3.1, if a particle $\overline{\mathbf{Q}}$ is, relatively to another one, **P**, an antiparticle, then in its own frame **Q** is a particle and $\overline{\mathbf{P}}$ is an antiparticle. Now, in an isolated system with absolute temperature T, the number of material particles with energy E_i is, according to Maxwell-Boltzmann distribution, given by [4]

$$n_i = A.e^{-E_i/k.T}.$$

If we accept that this equation remains valid to a system of antiparticles, it results that in this system the absolute temperature must be negative. In the case of the paraframe S^*,

$$T^* = -T.$$

As a matter of fact, the number n_i of antiparticles in S^* with energy $E_i^* = -E_i$ must be exactly the same then the one calculated for S (they are the *same* particles). Therefore, **antimatter must have a distinctive characteristic of negative absolute temperatures.** And, so, the barrier of **absolute zero**, like the barrier of the speed of light, appears to be a sort of "zero" unreachable for material or antimaterial particles. Remark that negative absolute temperatures are perfectly compatible with negative energies.

This reflexive "mirror characteristic" also applies to the *variation of* ***entropy***. The natural tendency of a system of particles to a statistic equilibrium – this means, to a partition of maximum probability – must also occur in any pseudotachyonic frame S^*. The point is that this tendency occurs in a positive time flow in S^*; but then, the time inversion in its paraframe *S'*, implies that the entropy of the correspondent system of antiparticles (which is also positive and equal to the one measured in the paraframe S^*) tends to conserve or to diminish. This statement corresponds to an **inversion of the 2nd law of Thermodynamics** in what concerns antimatter. It may also constitute an important criterion to decide if something observed in the Universe (a certain process or event) corresponds to matter or antimatter. And, of course, it is too a criterion to validate or not the present theory.

In trivial terms, to watch interactions between antimatter is like seeing a movie running backwards; and one should say: *"It may sound fantastic but, if an apple falls down from the tree, like Newton observed, we should observe an anti-apple returning to the anti-tree!"* We can go further in a cosmological speculation. In the first place, if we admit that time for antimatter flows inversely to our own (and vice-versa, evidently) then in the Universe it must coexist **two opposite time flows!** Secondly, the two fundamental *"physical realities"* mentioned in section 4.1 are certainly related to these two opposite time flows. According to the principle of relativity, one should not favour one time arrow instead of the other, their relationship being reciprocal. Indeed, for that global antimaterial frame, it is *our* space the one that is made up of antimatter and in which time flows backwards. And so, our *future* corresponds to the *past* of the antimatter. Should this mean that in our far future a *Big Crunch*, an inverted *Big Bang*, will take place? It is a good hypothesis!...

As mentioned in the final of section 2, a pseudotachyonic Field Theory (still in progress) clearly shows that negative mass/energy implicates *repulsive gravity*. Cosmologically, of course, this *antigravity* assumes an enormous importance in the evolution of the Universe. Usually, the scientific community presupposes that gravity is always attractive and, therefore, the Universe could only count upon its expansive energy to oppose to the omnipresent intrinsic gravity. If no one can take for granted that the Universe will not fall into an hopeless collapse, this canonical point of view has been the cause of serious difficulties in the adjustment of theories to the astronomical reality. Whilst hypothetical *dark matter* constitutes an enigma, some results in the observation of supernovae billion light-years distant from us (*Hogan, Kirshner and Suntzeff*) suggest that, contrary to the general opinion, the expansion of the Universe seems to be accelerating. The due consideration of the repulsive antigravity generated by all the antimatter that may exist in the Universe shall possibly enable a fresh insight to these and other cosmological problems and the drawing of a new picture. I certainly hope so.

Remark that, since there is any difference in the *nature* of homologous particles, we must admit that their creation cannot favour one of them. As a matter of fact, according to Dirac (and this theory agrees with him), creation must be simultaneous, in pairs. Could a violent gravitational separation between matter and antimatter have occurred in the origin of the Universe (if there was one), thus explaining the fact that Antimatter doesn't abound in our observable world of Matter? If so, a likely hypothesis consists in the admission of a symmetrical expansion of space-time; but then, the two polarized portions of the Universe may still continue to mutually repel.

Anyway, the separation of matter and antimatter worlds, which may also occur in "clusters", avoids severe causality problems to arise constantly. In whatever material frame we consider, this kind of 'avoiding property' lies in the fact that local antimaterial particles, if they are in minority, usually don't last too long.

8 Conclusion

We have extended the pseudotachyonic theory to energy and other fundamental variables, establishing a conception of antiparticle as the detection, with negative energy, of a tachyonic homologous particle. This includes antiphotons, which behave differently from photons.

An explanation has been presented for the universality of charge quantization and Compton effect has been generalized to antiparticles. We studied the pair annihilation/creation problem, with some new conclusions although its general comprehension is still open to investigation. Causality laws (like the 2nd law of thermodynamics) appear inverted for antimatter, which may be characterised by negative absolute temperatures.

In a forthcoming article, we'll treat the question of De Broglie waves – which, for ordinary particles, are tachyonic – re-enabling their real existence, looking for their physical signification and dethroning the mighty wave packets of the standard models. We'll then establish the premises for a pseudotachyonic Field Theory.

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Comment on ANTIMATTER AND PSEUDOTACHYONIC RELATIVITY

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It is rather unlikely that the physics community will accept the approach presented by L. Ferreira in his paper. The paper is however a beautiful illustration of the fact that an exciting idea (to view antiparticles as traces of tachyons) may fail if one tries to incorporate it into traditional standard frame. I am deeply convinced that completely new ideas may be developed only after drastic change of existing theories.

All extensions of Special Theory of Relativity always stumble on the notion of proper time τ related to physical time by the relation

$$d\tau = dt\sqrt{1 - \frac{\overrightarrow{v}(t)^2}{c^2}}. \tag{1}$$

where $\overrightarrow{v}(t)$ is the velocity of the moving body the motion of which is investigated. Clearly, it is the square root which restricts the velocities of motion to the domain

$$|\overrightarrow{v}(t)| \leq c. \tag{2}$$

The appearance of the proper time comes from the generalization of the Galilean invariant Newton equations

$$\frac{d\overrightarrow{x}(t)}{dt} = \overrightarrow{v}(t), \tag{3}$$

$$\frac{d\overrightarrow{p}(t)}{dt} = \overrightarrow{F}(t) \tag{4}$$

to Lorentz invariant equations

$$\frac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau), \tag{5}$$

$$\frac{dp^{\mu}(\tau)}{d\tau} = F^{\mu}(\tau). \tag{6}$$

The above and commonly used passage from Galilean-Newton mechanics to Einstein one has at least two serious disadvantages:

1) the parameter τ is not a monotonically increasing function of physical time t, especially for periodic motions,

2) the scheme cannot be generalized to many body systems where each particle has its own proper time.

Fortunately enough, there exists another, quite different, way of passing from the Galilean to Einsteinian mechanics in which the proper time never appears. In fact, let us first rewrite eq. (3) in the form

$$d\overrightarrow{x}(t) = \overrightarrow{v}(t)\,dt \tag{7}$$

and then use its relativistic analog in the form

$$dx^{\mu} = V_{\nu}^{\mu}(x)\,dx^{\nu}, \tag{8}$$

where $V_{\nu}^{\mu}(x)$ is some tensor field which we shall call the velocity tensor. In the Galilean case this tensor depends only on the time variable and in the matrix representation is given as

$$V(t) = \begin{pmatrix} 1, & 0, & 0, & 0 \\ v_x(t), & 0, & 0, & 0 \\ v_y(t), & 0, & 0, & 0 \\ v_z(t), & 0, & 0, & 0 \end{pmatrix}. \tag{9}$$

It is easy to check that (9) indeed represents a Galilean tensor and using (9) in (8) we immediately get (7).

In general, (8) is an eigenequation for the matrix $V(x)$ with eigenvalue 1. We want to have this eigenvalue as the only non-zero and non-degenerated eigenvalue. Otherwise, the relation (8) will not uniquely determine the eigenvector $\left(dx^0, dx^1, dx^2, dx^3\right)$ and the motion will not be uniquely described. In order to fulfill this requirement, the characteristic equation for the eigenvalues λ of the matrix V :

$$\begin{array}{c}(-\lambda)^n + (-\lambda)^{n-1} Tr_1 V + (-\lambda)^{n-2} Tr_2 V + ... \\ + (-\lambda)^2 Tr_{n-2} V + (-\lambda) Tr_{n-1} V + Tr_n V = 0\end{array} \tag{10}$$

must reduce to the simple form

$$\lambda^{n-1} (\lambda - 1) = 0. \tag{11}$$

Here, n is the dimension of spacetime and $Tr_j V$ are the sums of diagonal minors of the dimension j. Clearly, $1 \leq j \leq n$ and

$$Tr_1 V = Tr V, \qquad Tr_n V = \det V. \tag{12}$$

The eigenvalues $\lambda = 0$ all correspond to the rest of the bodies.

From (11) and (10) the matrix V must obey the conditions

$$\begin{array}{c}Tr_1 V = 1, \\ Tr_j V = 0,\end{array} \tag{13}$$

for $2 \leq j \leq n$. Unfortunately, it is not easy to solve these conditions for arbitrary n. But for $n = 2$ it can be done explicitly because in this case the two-dimensional matrix V must satisfy only two conditions

$$Tr V = 1 \tag{14}$$

and

$$\det V = 0. \tag{15}$$

It is easy to check the matrix

$$V(x) = \begin{pmatrix} 1 - k(x)\beta(x), & k(x) \\ [1 - k(x)\beta(x)]\beta(x), & k(x)\beta(x) \end{pmatrix} \tag{16}$$

indeed satisfies conditions (14). Here $k(x)$ is some arbitrary function of spacetime variables while $\beta(x)$ was introduced in order to get from (8) the relation

$$dx^1 = \beta(x) dx^0, \tag{17}$$

which identifies $\beta(x)$ with the ordinary velocity in (7).

Since we want to have the velocity tensor depending only on the ordinary velocity $\beta(x)$ we shall further assume that the function $k(x)$ in (16) is a function of the velocity $\beta(x)$, i.e.:

$$k(x) = K(\beta(x)) . \tag{18}$$

It turns out that the shape of this function can be fixed from the transformation rule of the velocity tensor

$$V^{\mu}_{\nu}(x) \to V^{\mu'}_{\nu'}(x') = \frac{\partial x^{\mu'}}{\partial x^{\alpha}} V^{\alpha}_{\beta}(x) \frac{\partial x^{\beta}}{\partial x^{\nu'}} \tag{19}$$

under the change of coordinates $x^{\mu} \to x^{\mu'}$. The explicit form of these transformation rule depends on the symmetries of spacetime. In the Galilean case from (16), (17) and (18) we get the functional equation

$$K(\beta - \beta_r) = K(\beta) , \tag{20}$$

where β_r is the relative velocity of two inertial systems of reference. Clearly, eq. (19) means that $K(\beta)$ is a constant. In the Newtonian mechanics this constant vanish.

In the Minkowski spacetime from (16), (17) and (18) we get the functional equation

$$K\left(\frac{\beta - \beta_r}{1 - \beta\beta_r}\right) = \frac{\beta_r(1 - \beta\beta_r)}{1 - \beta_r^2} + K(\beta)\frac{(1 - \beta\beta_r)^2}{1 - \beta_r^2} \tag{21}$$

with the solution

$$K(\beta) = \frac{\kappa - \beta}{1 - \beta^2}, \tag{22}$$

where κ is an arbitrary constant. The relativistic velocity tensor has therefore the form

$$V(t) = \frac{1}{1 - \beta(t)^2}\begin{pmatrix} 1 - \kappa\beta(t), & \kappa - \beta(t) \\ [1 - \kappa\beta(t)]\,\beta(t), & [\kappa - \beta(t)]\,\beta(t) \end{pmatrix}. \tag{23}$$

It is seen that this expression has a singularity at $\beta(t)^2 = 1$. This singularity, however, does not forbid to consider the case when $\beta(t)^2 > 1$.

To formulate the dynamical laws we may use the fact that the only generally covariant equation, similar to the Newtonian equation (4), has the form

$$\partial_\mu \pi^{\mu\nu}(x) = F^\nu(x), \tag{24}$$

where $\pi^{\mu\nu}(x)$ is an antisymmetric tensorial density and $F^\nu(x)$ is the density of force.

The next step is the use of the constitutive relation

$$\pi^{\mu\nu}(x) = M^{\mu\rho}(x)\, V_\rho^\nu(x) - M^{\nu\rho}(x)\, V_\rho^\mu(x) \tag{25}$$

which is the analogy of the non-relativistic relation

$$\overrightarrow{p}(t) = m\overrightarrow{v}(t). \tag{26}$$

Assuming that the mass tensor $M^{\mu\rho}(x)$ is a function of the velocity β and proceeding exactly as above from the relativistic transformation rule of $M^{\mu\rho}(x)$ we get the following general shape of the mass tensor

$$\begin{aligned}
M^{00}(\beta) &= \tfrac{1}{1-\beta^2}\left(M^{00} - \beta M^{01} - \beta M^{10} + \beta^2 M^{11}\right)\\
M^{01}(\beta) &= \tfrac{1}{1-\beta^2}\left(M^{01} - \beta M^{00} - \beta M^{11} + \beta^2 M^{10}\right)\\
M^{10}(\beta) &= \tfrac{1}{1-\beta^2}\left(M^{10} - \beta M^{11} - \beta M^{00} + \beta^2 M^{01}\right)\\
M^{11}(\beta) &= \tfrac{1}{1-\beta^2}\left(M^{11} - \beta M^{01} - \beta M^{10} + \beta^2 M^{00}\right).
\end{aligned} \tag{27}$$

As it is seen there is no square root $\sqrt{1-\beta^2}$ either but there is a singularity at $\beta^2 = 1$. This singularity can be however removed by the choice

$$\begin{aligned}
M^{11} &= -M^{00},\\
M^{10} &= -M^{01}.
\end{aligned} \tag{28}$$

In this case the mass tensor has the simple form

$$M = \begin{pmatrix} M^{00} & M^{01} \\ -M^{01} & -M^{00} \end{pmatrix} \tag{29}$$

and the inertial properties of bodies are described by two mass parameters.