



ANTIGRAVITY : A CRAZY IDEA ?

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ABSTRACT :

The theoretical aspect of Antigravity is briefly discussed. It is shown that supergravity with  $N = 2, 3, \dots, 8$  fermionic generators leads naturally to Antigravity.

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Let us consider, between two particles, the tree diagram due to the exchange of a massless graviton, and of a massless vector field  $A_\mu^{\lambda}$ , which we shall call the antigraviton. The coupling of these two fields to matter fields  $\phi^i$  (scalars), or  $\chi^k$  (Dirac spinors) is given by :

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4\kappa^2} V V^{\mu\alpha} V^{\nu\beta} R_{\mu\nu\alpha\beta} - \frac{1}{4} V g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu}^{\lambda} F_{\rho\sigma}^{\lambda} \\ & + V (g^{\mu\nu} \sum_i (\partial_\mu \phi^i)^* (\partial_\nu \phi^i) - m_i^2 \phi^{*i} \phi^i) \\ & + V \sum_k \bar{\chi}^k (i \gamma^\mu \overleftrightarrow{\partial}_\mu - m_k) \chi^k \end{aligned} \quad (1)$$

where we set :

$$F_{\mu\nu}^{\lambda} = \partial_\mu A_\nu^{\lambda} - \partial_\nu A_\mu^{\lambda} \quad (2)$$

and

$$\partial_\mu \phi^j = \partial_\mu \phi^j - i g_j A_\mu^{\lambda} \phi^j \quad (3)$$

$$\partial_\mu \chi^k = \partial_\mu \chi^k - i g_k A_\mu^{\lambda} \chi^k \quad (4)$$

The vector  $A_\mu^{\lambda}$  is coupled to a conserved U(1) current  $\partial_\mu j^{\lambda}$  and the charges  $g_i, g_k$  are a priori unrelated. The one-boson exchange graph is given by :

$$\mathcal{A} = \frac{8\pi G}{q^2} \left[ T_{\mu\nu} T_{\mu\nu}' - \frac{1}{2} T_{\mu\mu} T_{\nu\nu}' - \frac{g g'}{8\pi G} \partial_\mu j^{\lambda} \partial_\mu j^{\lambda}' \right] \quad (5)$$

In the static limit, setting  $\kappa^2 = 4\pi G$ , we find, for two particles of masses  $m, m'$ , of charges  $g, g'$

the formula :

$$A = \frac{4\kappa^2 m m'}{g^2} \left[ m m' - \frac{g g'}{4\kappa^2} \right] \quad (6)$$

The first term is always positive (gravitational attraction).

The sign of the second term is negative, hence repulsive if we have two particles or two antiparticles (assume that  $g_i > 0$  for all particles,  $\bar{g}_i = -g_i$  for an antiparticle,  $g_i = 0$  for a self-conjugate particle under  $C$ , such as the  $\gamma$ , the gluons, etc...), and positive, hence attractive, between a particle and an antiparticle.

We shall call antigravity the phenomenon which occurs if the net force (gravity + antigravity) is zero between any two particles.

This implies the universal formula :

$$g_i = 2\kappa m_i \quad (7)$$

Let us first see if this is ruled out. As two neutrons attract each other, it seems that, immediately, antigravity must be eliminated. However, the masses which appear in (1) are the quark and lepton masses, not those of the proton and neutron. Indeed, the antigraviton couples to  $e^-$ ,  $u$ ,  $d$ , etc.. and sees their bare mechanical mass, since  $\int \delta^R$  is conserved. In a composite particle, such as  $p$ , it does not see the gluons and its coupling to a proton is really given by :

$$\int \delta^R(p) = 2\kappa (m_u \bar{\Psi}_u \gamma^\mu \Psi_u + m_d \bar{\Psi}_d \gamma^\mu \Psi_d) \quad (8)$$

while the graviton is coupled to the real mass (it sees the gluons).

As a result, the force between 2 atoms  $(Z, A)$ ,  $(Z', A')$  is given by :

$$F = \frac{8\pi G}{r^2} [M M' - M^0 M^{0'}] \quad (9)$$

where (footnote 1)

$$M = Z (M_p + m_e) + (A - Z) M_n \quad (10)$$

$$M^0 = Z (2m_u + m_d + m_e) + (A - Z) (m_u + 2m_d) \quad (11)$$

For the Earth, we can replace  $\frac{M_{EARTH}^0}{M_{EARTH}}$  by  $\frac{3m_u}{M_p}$

Deviations from the equivalence principle occur, and one finds that the acceleration of two atoms  $(Z, A)$ ,  $(Z', A')$  towards the Earth differs by

$$\frac{\delta\gamma}{\gamma} = (Z'A - ZA) \frac{3m_u}{M_p} \times \frac{\mu^2}{M M'} \quad (12)$$

where

$$\mu^2 = m_e (M_n - m_u - 2m_d) + \frac{3}{2} (m_u + m_d) (M_n - M_p) + \frac{1}{2} (m_u - m_d) (M_n + M_p) \quad (13)$$

In the case of exact  $SU(2)$  symmetry (which is probably wrong anyhow), setting  $m_u = m_d$ , we get :

$$\mu^2 \sim m_e M_n \quad \text{with} \\ M_n - M_p \sim 1.7 \text{ MeV} . \text{ Setting } m_u = 10 \text{ MeV (this is highly debatable), we get :}$$

$$\mu^2 \sim 500 (\text{MeV})^2$$

and 
$$\frac{\delta\gamma}{\gamma} \sim 1.5 \cdot 10^{-5} \left( \frac{Z'}{A'} - \frac{Z}{A} \right) \quad (14)$$

This is clearly bad news for antigravity, since the Eötvos experiment gives  $\frac{\delta\gamma}{\gamma} < 10^{-9}$  and R.H. Dicke pushed the limit down to  $10^{-11}$  [1]

The situation can be saved, however, if one of the scalar fields acquires a non-zero vacuum expectation value, as in the breaking of  $SU(2) \times U(1)$  down to  $U(1)$ . Then the  $\lambda$  acquires a mass given by

$$m_\lambda = 2\kappa m_\phi \langle \phi \rangle \quad (15)$$

The reason why  $\phi$  acquires a v.e.v. while  $m_\phi^2 > 0$  can be due to radiative corrections (of order  $g^2_{\text{strong}}, \alpha$  or  $\kappa^2$ ) which can turn a potential having a minimum at the origin into one which has a maximum, the true minimum being elsewhere [2]. To fix ideas, we can set  $m_\phi \sim 1 \text{ GeV}$ ,  $\langle \phi \rangle \sim 1 \text{ GeV}$ , which gives to the  $\lambda$  a tiny mass  $m_\lambda \simeq 10^{-19} \text{ GeV}$ , or a Compton wavelength  $R_\lambda = \frac{1}{m_\lambda} \sim 1 \text{ Km.}$  (footnote 2)

In this case Antigravity is saved, since the potential is given by

$$V = \frac{8\pi G}{r} \left[ M M' - \exp - \frac{r}{R_\lambda} M^\circ M'^\circ \right] \quad (16)$$

On the surface of the Earth one can safely replace

$\exp - \frac{R_{\text{EARTH}}}{R_\lambda}$  by 0 so that the Antigravity contribution is negligible.

To realize an Antigravity device based on this idea would be cumbersome, but, in theory, possible: one would have to

"heat up the vacuum" to reach the phase where  $\langle \phi \rangle = 0$ , but also the one where the quarks are free so that  $M_p \equiv 2m_u + m_d$  !  
 $M_m = 2m_d + m_u$ , which may be a real disaster for the space-ship. At this cost it might be possible if we know how to "heat up the vacuum" without destroying the engine. However, this clearly belongs either to UFOlogy [3] or Science-Fiction [4] not yet to Technology.

Now what about the strange relation  $g_i = 2\kappa m_i$ ? This universal formula clearly comes from the sky. However, in extended ( $N = 2, 3, \dots, 8$ ) supergravity theories, one can show that there is always a massless vector field  $A_\mu^\lambda$  which couples precisely with strength  $\pm 2\kappa m_i$ , whatever the field it is coupled to ( $J = 3/2$  or 1, or  $1/2$  or 0). In the  $N = 2$  case, this was deduced purely from the algebra of two supersymmetry transformations [5] and thus it was guessed [6] (but unproved) that  $N = 2$  supergravity led to antigravity. In 1977 K. Zachos worked out the coupling of  $N = 2$  supergravity ( $V_\mu^r, \Psi_\mu^i, A_\mu^\lambda$ ) to a massive matter multiplet ( $\chi^i, \phi^i$ ) and discovered the phenomenon of antigravity [7]. In the case  $N = 3, \dots, 8$  it is now known that antigravity also occurs and the mysterious formula (8) holds for all 256 states of the  $N = 8$  theory [8] [9].

In addition, the formula  $|g| = 2\kappa m$  is not so mysterious if one realizes that the  $N = 8$  theory with 4 mass parameters is obtained by dimensional reduction from 5 dimensions to 4. If we exchange a massless graviton in 5 dimensions between 2 massless particles of momenta  $p_1^{\hat{\mu}}, p_2^{\hat{\mu}}$  in the static limit we find the amplitude given by

$$\mathcal{A} = \frac{8\pi G}{9^2} (p_1 \cdot p_2)^2 \quad (17)$$

Setting

$$p_1^{\hat{\mu}} = m_1 (1, 0, 0, 0, \epsilon_1) \quad (18)$$

$$p_2^{\hat{\mu}} = m_2 (1, 0, 0, 0, \epsilon_2) \quad (19)$$

where  $\epsilon_i = +1$  for a particle,  $-1$  for an antiparticle (by convention), we get :

$$\mathcal{A} = \frac{8\pi G}{9^2} m_1^2 m_2^2 (1 - \epsilon_1 \epsilon_2)^2 \quad (20)$$

The cancellation of forces is obvious for 2 particles or 2 antiparticles. The relation  $(p_{\hat{\mu}})^2 = 0$  in  $D = 5$  translates in  $D = 4$  into

$$p^2 - m^2 = 0 \quad (21)$$

This phenomenon of cancellation of forces is known to occur also in vector scalar systems, both for classical fields [10] and magnetic monopoles [11] and has the same interpretation [12]. The  $N = 8$  theory with 4 mass parameters [9] is of particular interest. In the limit where  $m_{1,2,3} = m$  and  $m_4 = M$ , it has an  $SU(3) \times U(1)$  invariance. At zero mass, it has a graviton, 8 gluons of electric charge  $Q = 0$ , and 2 vectors, one which is identified with the photon ( $\gamma$ ), and one which is the antigraviton ( $\lambda$ ) and gauges the charge  $g = \pm 2\kappa m$  for all states of the theory and thus can hardly be a  $Z^0$ . In addition, the model contains a  $d$  quark of mass  $2m$ , an

electron of mass  $3m$ , a  $u$  quark of mass  $M - 2m$ , 8 gluinos of mass  $M$ , a  $c$  quark of mass  $M + 2m$ , as well as a triplet of  $d$ -type ( $Q = -1/3$ ) gravitinos of mass  $m$ , and a singlet gravitino of charge 0 and of mass  $M$ . Although poor in leptons and quarks, this might be a model for a future unified theory of gravity + matter, and predicts antigravity. If the  $SU(8)$  local invariance discovered by Cremmer and Julia [13] in the massless case leads to the appearance of  $SU(8)$  gauge fields through bound states, it might even be a realistic grand-unified theory [14].

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## FOOTNOTES :

- (1)  $M_p, M_n$  should really read  $M_p - \Delta E, M_n - \Delta E$  where  $\Delta E$  is the nuclear binding energy per nucleon. This modifies slightly the proton and neutron masses which we shall, for the sake of simplicity, take to be equal both to 1000 MeV. Similarly, the ratio  $m_u/M_p$  will be small ( $10^{-2}$ ).
- (2) This can also occur classically if  $\langle \Phi \rangle \neq 0$ , provided that in the original Lagrangian,  $A_\mu^2$  couple with strength  $\pm 2K|\mu|$  even when  $\mu^2 \leq 0$ . We thank E. Cremmer for pointing this out.