SHARK ATTACK!

ANON

Let C be a monoid such that

- there exists a group G such that $C \leq G$, i.e. C is a submonoid of G when G is considered as a monoid
- for all $c, c' \in C$, if $cc' = 1$, then $c = c' = 1$.

Then C is a **sector**. Let C, D be sectors and $e\colon C \to D$ a map such that $e(1) = 1$ and $e(cC) \subseteq e(c)D$ for all $c \in C$. Then e is a **calc**. Let $\mathrm{Calcs}(C, D)$ be the set of calcs with domain C and codomain D.

For each element $c \in C$, let $f = e^c$ be the unique map $f\colon C \to D$ such that $e(c)f(c') = e(cc')$. It is left as an exercise to the reader to prove that this map exists, is unique, and is a calc. This is the **shift** of e by c. The sector C acts on the right on both $e^C = \{e^c\colon c \in C\}$, the set of shifts of $e$by an element of $c \in C$, and $\mathrm{Calcs}(C, D)$ by shift.

Let $\mathcal{C}$ be a nonempty collection of sectors and $\mathcal{D} \subseteq \mathcal{C}$ a nonempty set. Let $S\colon \mathcal{D} \to \mathcal{P}_\omega(\mathcal{C})\backslash\emptyset$ be a map from $\mathcal{D}$ to the collection of finite nonempty subsets of $\mathcal{C}$. An S**-fin** taking values in $D \in \mathcal{D}$ is a calc

$$f\colon \prod_{C \in S(D)} C \to D.$$

Let $\mathrm{Fins}(S, \mathcal{D})$ be the union of the sets of S-fins taking values in some $D \in \mathcal{D}$. A map $\mathcal{F}\colon \mathcal{D} \to \mathrm{Fins}(S, \mathcal{D})$ such that $f_D = f_{1D} = \mathcal{F}(D)$ is an S-fin for taking values in D is an $(S, \mathcal{D})$**-shark**, an indexed collection of S-fins.

Given an element $\overline{c} \in \prod_{C \in \mathcal{C}} C$ of **phase space**, the shark's natural inclination is to feed on the projection of $\overline{c}$ onto the coordinates $S(D)$, i.e. $\pi_{S(D)}(\overline{c})$, and the fins tell us how each coordinate is updated. Those coordinates $C \in \mathcal{C}\backslash\mathcal{D}$ are not updated, and for $D \in \mathcal{D}$, we have

$$c_D = \mathcal{F}(D)(\pi_{S(D)}(\overline{c})).$$

According to "calc-shift karma-vipāka" the feeding shifts each calc in the shark $\mathcal{F}$ to $\mathcal{F}^{\overline{c}}$, i.e.

$$\mathcal{F}^{\overline{c}}(D) = \mathcal{F}(D)^{\pi_{S(D)}(\overline{c})}.$$

Note that for sectors C, D and a calc $e\colon C \to D$, the domain C acts on the right on both e^C, the set of shifts of e by an element $c \in C$, and $\mathrm{Calcs}(C, D)$, the set of calcs with domain C and codomain D, by shift. So too does an element $\overline{c} \in \prod_{C \in \mathcal{C}} C$ act on the right on $\mathcal{F}^{\prod_{C \in \mathcal{C}} C}$ and the set of $(S, \mathcal{D})$-sharks by shift, as described above.

The **shark attack** started at $\overline{c} = \overline{c}_1 = (c_{1C})_{C \in \mathcal{C}}$ consists of feeding, where each coordinate value c_C is fed to fin inputs, producing feed argument elements

$$\pi_{S(D)}(\overline{c}) \in \prod_{C \in S(D)} C$$

where $\pi_{\mathcal{E}} \colon \prod_{C\in\mathcal{C}} C \to \prod_{C\in\mathcal{E}} C$ is the coordinate projection map, and feed value elements

$$c_{2D} = f_D(\pi_{S(D)}(\overline{c}_1)),$$

yielding survivors

$$c_{2C} = \begin{cases} c_{2D} & \text{when } C = D \in \mathcal{D} \\ c_{1C} & \text{otherwise} \end{cases}$$

and happy shark $f_{2D} = f_{1D}^{\pi_{S(D)}(\overline{c}_1)}$ where $f_{1D} = f_D$. Feeding continues indefinitely according to

$$c_{n+1C} = \begin{cases} f_{nC}(\pi_{S(C)}(\overline{c}_n)) & \text{when } C \in \mathcal{D} \\ c_{nC} & \text{otherwise} \end{cases}$$

and

$$f_{n+1D} = f_{nD}^{\pi_{S(D)}(\overline{c}_n)}.$$

The **feed sequence** is $(\overline{c}_n)_{n=1}^{\infty}$, and the **shark sequence** is $(f_n)_{n=1}^{\infty}$.

Markov Property

Let C, D be sectors and $\mathcal{A}$ a σ-algebra on Ω, a probability measure space with probability measure μ. Let $\mathcal{B}$ be a σ-algebra on $\mathrm{Calcs}(C, D)$. A random calc with domain and codomain C is given as an $(\mathcal{A}, \mathcal{B})$-measurable map $E \colon \Omega \to \mathrm{Calcs}(C, D)$, and we have $P(E \in \beta) = \mu(E^{-1}(\beta))$ for $\beta \in \mathcal{B}$.

Let $t \in C$ be a nonidentity element called the **test**. If $(E^{c_n}(t))_{n=1}^{\infty}$ is a markov chain for all prefix monotone sequences $\overline{c} = (c_1, \ldots)$ in C, so for all n, there exists some $\delta_n \in C$ such that $c_n\delta_n = c_{n+1}$, then E is a **markov atom** with test t.