Let A be a monoid and B a submonoid. The element $a \in A$ omits B if for all $b \in B$ and $u, v \in A$,

$$ubv = a \implies b = 1,$$

and the submonoid C omits B if c omits B for all $c \in C$.

Let A be a monoid and B, C submonoids, and suppose that for all $a \in A$, a omits B whenever a omits C. We say that B is **absolutely continuous** with respect to C and write $B \ll C[1]$.

Exercise. Let A be a monoid and $B, C \leq A$. Prove that if $B \leq C$, then $B \ll C$.

Exercise. Find a monoid A and submonoids $B, C \leq A$ such that $B \not\leq C$ and $B \ll C$. (Hint: Let $A = \langle x, y \colon xy = x \rangle$.)

References

[1] "Real Analysis" Folland (1984) p. 83.