Let A be a monoid and $B, C \leq A$ submonoids such that $B \cap C = \{1\}$ and $\langle B \cup C \rangle = A$. If the binary relation

$$\{(\prod_{i=1}^{n} b_i c_i, \prod_{i=1}^{n} b_i) \colon \bar{b} \in B, \bar{c} \in C\} = \pi_B^C \subseteq A \times A$$

is a function $\pi_B^C \colon A \to B$, then it is the B**-filter along** C. It is left as an exercise for the reader to prove that it is an idempotent homomorphism when it exists. If $\pi_B^C = \pi_B^{C'}$ for all $C' \leq A$ such that $B \cap C' = \{1\}$ and $\langle B \cup C' \rangle = A$, then it is π_B, the B**-filter**.

References

[1] Folland "Real Analysis" (1984) p. 82.