Example. Galois group of a right regular differential action. Let $E = \{e_1, e_2, \ldots\}$ be an infinite set disjoint from the natural numbers, and let $\mathbb{N}$ act on E on the right by $e_n k = e_{n+k}$. Recall $E \rtimes \mathbb{N} = \{(e, ek)\colon e \in E \wedge k \in \mathbb{N}\}$ and let $d\colon E \rtimes \mathbb{N} \to \mathbb{N}$ be the differential action $d(e, ek) = k$. Let $A \leq \mathbb{N}$ be a submonoid of the natural numbers under additon and $F \subseteq E$ any subset. The **galois group** for A with fixed set F is

$$\begin{gathered}\mathrm{Aut}_A(E|F) = \{\sigma \in S_E\colon \sigma(e+k) = \sigma(e) + k \wedge \\ d(e, e+k) = d(\sigma(e), \sigma(e+k)) \\ \forall e \in E\, \forall k \in A \wedge \sigma f = f\, \forall f \in F\}.\end{gathered}$$

The action of $\mathbb{N}$ on E is the right regular differential action of the identity map on the natural numbers.

In general, Let C be a monoid and X a right C-set. A **differential** taking values in a monoid D is a map $d\colon X \rtimes C \to D$ such that $d(x, x) = 1$ and $d(x, xc)d(xc, xcc') = d(x, xcc')$ for all $x \in X$ and $c, c' \in C$.

Let C, D be sectors and $e\colon C \to D$ a calc. The **right regular differential action** for e is the right regular monoid action of C on itself plus the differential $d(c, cc') = e^c(c')$. The map e^c is the **shift** of e by an element $c \in C$, the unique calc $f\colon C \to D$ such that

$$e(cc') = e(c)f(c').$$

A **sector** is a group-embeddable monoid C such that $cc' = 1 \implies c = c' = 1$ for all $c, c' \in C$. For sectors C, D, a map $e\colon C \to D$ is a **calc** when $e(1) = 1$ and $e(cc') \in e(c)D$ for all $c, c' \in C$.