Regarding the difficulty of Birch and Swinnerton-Dyer and its relevance to elliptic curves, Johnson writes the following [101].

> "There is no doubt that elliptic curves are amongst the most closely and widely studied objects in mathematics today. ***The arithmetic complexity of these particular curves is absolutely astonishing*** [*emphasis added*], so it isn't surprising the Birch and Swinnerton-Dyer conjecture has been honored with a place amongst the Clay Mathematics Institute's famous Millennium Prize Problems. Although some great unsolved problems carry the benefit of simplicity in statement, this conjecture is not one of them. There even seems to be an aura of 'hardness' over the problem that keeps many from discovering the true beauty of the conjecture. [...] The Birch and Swinnerton-Dyer conjecture today remains, of course, unsolved and most mathematicians agree that new ideas will need to be developed to tackle the great problem. A proof will take a great deal of work and mathematical power."

The present problem regarding the elliptic curve application for $\hat{M}^3$'s equation requires a survey of some large volume of number theory. The work might far exceed the ordinary scope of a PhD problem.

Part II: Problems in Physics

The thesis problems in Part II are presented with less detail than the problem in Part I. These problems are mostly applications for the MCM and/or fractional distance analysis toward open problems in physics.

2 Period Doubling

This problem in mathematical physics is as described in the following excerpt from [96]. It concerns period doubling behavior in equations such as (2.1).

> "The original idea for a second number line such as that which appears in [*the MCM*]—chiros as opposed to the original number line: chronos—came about in a study of the period doubling cascades that arise in chaotic dynamics. For example, consider convective rolls in a finite, bounded volume of fluid heated from below. [*This physical system is described by*
>
> $$\ddot{x} + k\dot{x} - x + 4x^3 = A\cos(\omega t) \quad . \tag{2.1}$$

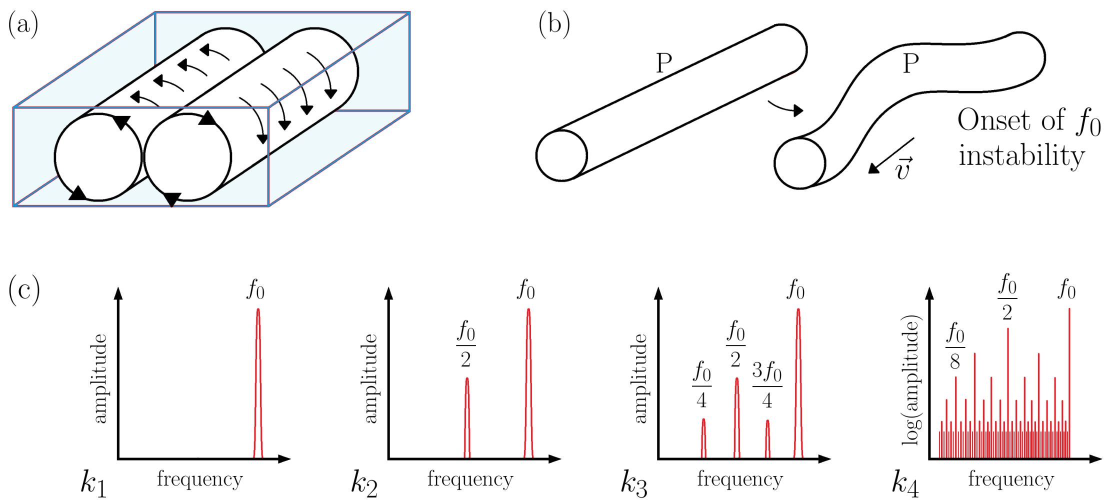

Figure 18: This figure is adapted from Cvitanović [102]. (a) Two stable, laminar convective rolls in a finite volume of fluid heated from below. (b) The development of an instability with frequency f_0 is shown. Waves will move along the axis of each cylinder with speed $|\vec{v}|$. In the laminar case, a temperature probe at P will show constant temperature. After the onset of first instability, the temperature at P will oscillate with frequency f_0 as the wave sweeps back and forth past the probe. (c) The period doubling cascade for four increasing values of k. As new modes of instability appear, the temperature at P will become increasingly erratic.

> *For low temperatures, heat convection in the fluid is laminar, as in Figure 18a.*] When the temperature gradient in the cell reaches a first critical value [*the laminar convective*] rolls will become unstable. [*Waves will*] begin to move along the rolls' axial direction with some frequency f_0 [*as in Figure 18b*]. As the heat increases, more waves will appear with frequency $f_0/2$[.] [*As*] heating increases more it will be possible to observe waves with frequency multiples of $f_0/4$, then $f_0/8$, etc., until period doubling exceeds the resolution of the experimental apparatus and eventually the onset of turbulence is complete [*as in Figure 18c.*]"

The dissipation parameter k in (2.1) is a mathematical representation of the heat flow through the fluid. As the heat at the bottom of the fluid volume continues to increase, a convective roll will acquire an instability moving axially with frequency f_0 (Figure 18b). It will become more unstable in a regular way until full turbulence eventually sets in. The second instability will be a second axial wave with frequency $f_0/2$. In the regime of second instability, one understands that two waves with different frequencies are superposed on the fluid cylinder. Recalling that period is inverse frequency, the period doubling cascade ends with the onset of turbulence associated with $f_0/2n$ as $n\to\infty$.

As k smoothly increases, the amplitude of existing instabilities increases. The nature of the present problem regards the transition from zero amplitude to non-zero amplitude as new modes of instability appear. Calling the amplitude associated with the n^{th} frequency mode A_n and using k_n to describe the value of k at which the n^{th} mode appears, we have

$$\lim_{k\to k_n^+} A_n(k) = 0 \quad . \tag{2.2}$$

One might think of adding k to Figure 18c in the direction perpendicular to the page so that each A_n appears as a long ridge terminating on a flat plane. As k is decreased back toward k_n after A_n has appeared, the amplitude of that instability must go to zero. An infinite number of $\partial_k^{(m)}A_n(k)$ derivatives must also go to zero as $k\to k_n^+$ because A_n does not exist for any $k<k_n$. In the estimation of this writer, this implies discontinuous behavior at k_n which cannot be derived by the smooth variation of k itself. For instance, the frequency peaks in Figure 18c should be described as Gaussians ϕ_n which have no zeros on the real line. It is not clear where one might insert k into

$$\phi_n(f) = A_n \exp\left\{\frac{\left(f-b\right)^2}{2c}\right\} \quad , \tag{2.3}$$

to affect

$$\lim_{k\to k_n^+} \phi_n = 0 \quad . \tag{2.4}$$

The equation from which A_n is derived, (2.1), is not piecewise defined, so we should not attempt to explain the discontinuous behavior in A_n with any piecewise solutions. Furthermore, there is no parameter in (2.1) other than k to which we might attribute the sudden onset of a non-zero A_n, unless there is another hidden parameter somewhere in the underbelly of mathematics.

By adding a hidden (abstract) parameter χ in a fourth orthogonal direction to the space of A, f, and k, we may trigger the onset of $A_n\neq 0$ where a curve parameterized in χ intersects the f-k plane. We would introduce conformal infinity functions $k_\lambda = \tan(\gamma k+\delta)$ such that a zero of ϕ_n at $k_\lambda=\pm\infty$ is moved to the intersection of k and χ at k_n. The main purpose of the second number line charted in χ will be to preserve information about the spacing of the k_n where conformal k_λ allows Gaussians to go to zero at the onset of new instability modes. In the conformal parameter, k cannot easily contain information about locations beyond infinity. The χ direction is necessarily hidden from the ODE, but this mimics the condition in the unit cell where quantum states in $\mathcal{H}$ know nothing about the bulk. Successive k_n should be like $\lim\chi^4_\pm \to 0$ at successive $\mathcal{H}$-branes. The work of the present period doubling thesis would seek to embed an equation such as (2.1) in an analogue unit cell such that the discontinuous onset of new instability modes is triggered by something like an $\mathcal{H}$-brane where x^0 and χ^4 intersect, or k and χ in the present case. As we already have the Schrödinger equation to describe quantum evolution from t_1 to t_2 but we have suggested another equation for $\hat{M}^3$, this period doubling thesis might search for entirely new ways of solving differential equations.

Extensive and mysterious *universality in chaos* [102] can be taken to suggest a hidden parameter such as χ. It is not currently known why unrelated systems show remarkably similar behaviors in their chaotic limits. The ubiquity of constants such as Feigenbaum's numbers [103–105] in disparate chaotic systems is evocative of the universal numerical scheme in the ontological basis that we have used to append the unit cell to $\mathcal{H}$.

3 Field Line Breaking

Classical electromagnetism does not allow the formation of electromagnetic waves detached from their sources as propagating flux loops. This is unfortunate because the formation of such waves is thought to be a real physical process.

EM field lines are the level sets of the $\mathbf{E}$ and $\mathbf{B}$ fields which satisfy Maxwell's equations. These fields cannot have cusps in their level sets. However, for loops of flux to break off from their sources as propagating waves, a level set must acquire a cusp at some point: an X-point. The lack of any mechanism for such a process is a grievous deficiency in classical electromagnetism. A resolution to this problem would have far reaching consequences in almost all areas of physics. As it is, the tangent vector to a level set of the $\mathbf{E}$ field points in the direction of $\mathbf{E}$, and it follows that $\mathbf{E}$ would point in two different directions at an X-point.

Figure 19 shows field lines near an X-point: before and after. One is able to visualize the intermediate step at which the field lines must cross or reach a cusp, but that configuration of field lines is not a solution to Maxwell's equations. As in the period doubling cascade, here we must appeal to some new method for solving differential equations. The cusps can be smoothed over with quantum mechanics, but we would like to develop a method for field lines to break in classical field theory. Referring to Figure 19, note that field lines break and then reconnect to an exactly mirrored line. This is similar to what happens at $\varnothing$ where χ^4_+ terminates in a singularity and then

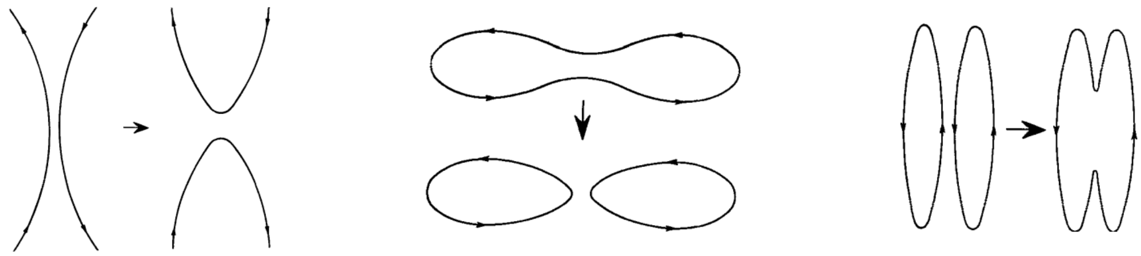

Figure 19: Figures due to McDonald [106] show EM field line configurations near X-points. The arrows show two cases of loops being formed and one case of two loops merging.

continues along χ^4_- on the higher level of aleph. Indeed, we have suggested specular reflection as a workaround for continuing χ^4 past the topological obstruction at $\varnothing$ (Section 1.6.6) [71].

The problem of field line breaking is similar to the problem of the forward connection of Σ^+ into Σ^-. It is not unlikely they are two sides of the same coin. Even the transition from topological AdS (the slices of Σ^-) to topological dS (the slices of Σ^+) exactly replicates the issue of field line breaking. The solutions of the AdS metric are hyperboloids of two sheets, and the solutions of the dS metric are hyperboloids of a single sheet (Figure 20). Between these spaces of uniform positive or negative curvature, the infinitely curved $\varnothing$-brane is like another X-point.[1] Even the pinching of the two classes of elliptic curve shown in Figure 17 (Section 1.11) is evocative of the same mysterious X-point.

Ideas for the utility of fractional distance analysis toward the X-point problem include the following. The arithmetic of numbers in the neighborhood of infinity may be related to field line breaking through loss of information about b in the $(\aleph_\mathcal{X}+b)/\widehat{\infty}=\mathcal{X}$ operation. If we were to incorporate levels of aleph into the solutions of differential equations, each time step being like an $\mathcal{H}$-brane, or some similar scheme, then two field lines passing through separate points $\aleph_\mathcal{X}+a$ and $\aleph_\mathcal{X}+b$ on one level of aleph would both pass through $\mathcal{X}$ on the higher level. Most generally, the identity $x/\infty=0$ for any $x\in\mathbb{R}_0$ is a hard constraint on field solutions in the neighborhood of the origin, but this constraint is relaxed in the neighborhood of infinity where new behaviors may be possible. Indeed, new behaviors are *implied* by the new arithmetic operations [2]. Furthermore, the strength of the **E** field where field lines cross is infinite, and $\widehat{\infty}$ is a new tool for dealing with infinite quantities. Similarly, $\mathbf{B}=0$ at X-

[1] Given a hyperboloid $x^2+y^2-z^2=\ell^2$, we have a circle of radius ℓ in the $z=0$ plane. Decreasing the hyperboloid parameter ℓ reduces the radius of the circle, so the infinitely curved $\ell^2=0$ case corresponds to the cusped intermediate conical case not pictured in Figure 20.

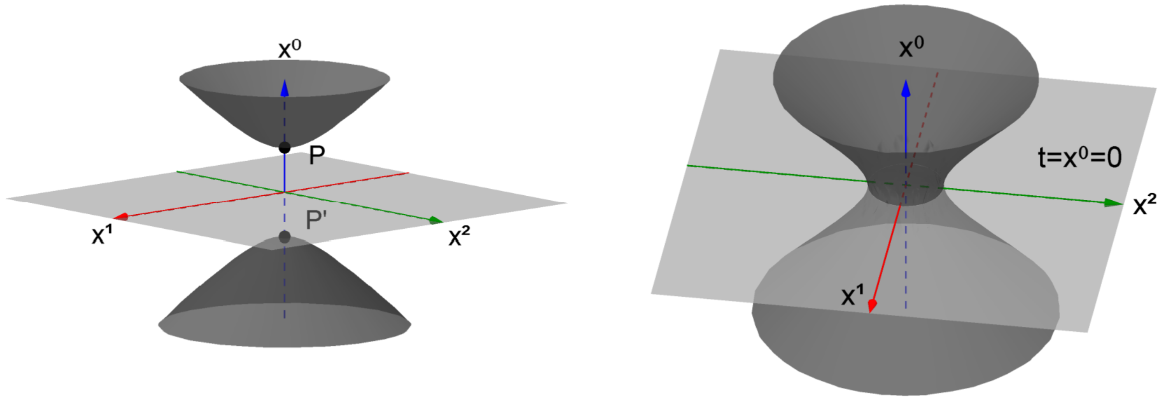

Figure 20: Figures due to Walker [107] show hyperboloids of one and two sheets. Anti-de Sitter space is associated with one of the two surfaces in a hyperboloid of two sheets (left) while de Sitter space is associated with a hyperboloid of one sheet. One may visualize an X-point associated with the flat geometry of $\mathcal{H}$ which separates the slices of Σ^- from the slices of Σ^+.

points, so we may study duality between $\widehat{0}$ and $\widehat{\infty}$ for applications toward descriptions of the combined EM field.[1] Another idea is to use the big part of $\aleph_\mathcal{X}+b$ as the usual position variable such that the little part functions as an effective infinitesimal. Such quasi-infinitesimals may be useful for describing physics near X-points.

4 Curvature in the Neighborhood of Infinity

This problem regards the $\Omega\to\mathcal{A}$ step of $\hat{M}^3$ as well as the identity and function of the $\varnothing$-brane. If $\varnothing$ is a topological singularity, we must determine what separates it from the regions of non-singular geometric curvature. To wit, fractional distance analysis has provided the non-arithmatic numbers (Section 1.6.6) such that we may place $\varnothing$ at $\mathcal{F}_\mathcal{X}\in\mathbb{F}$, but the fractional distance program has not uniquely determined whether a single number separates successive $\mathbb{R}_\mathcal{X}$ or if there exist intervals between them. Figure 21 shows these possibilities. The set of all $\{\mathcal{F}_\mathcal{X}\}$ might be totally disconnected or these numbers could be the midpoints of neighborhoods of non-arithmatics as the $\aleph_\mathcal{X}$ are the midpoints of the $\mathbb{R}_\mathcal{X}$ neighborhoods. Pertaining to the unit cell, we have not decided if a single point separates the Ω- and $\mathcal{A}$-branes, if there exists an interval

[1] Duality between $\widehat{0}$ and $\widehat{\infty}$ was detailed most specifically in [46]. Briefly, suppose there exists a Euclidean line segment AB covered by a chart $x\in[0,\infty)$ with 0 at A and ∞ at B. (AB is covered except for an endpoint.) For every $n\in\mathbb{N}$ in the neighborhood of the origin, the invariance of AB under permutations of the labels of its endpoints implies the existence of another number $\widehat{\infty}-n$ in the neighborhood of infinity. Duality between $\widehat{0}$ and $\widehat{\infty}$ means there exists a $\widehat{\infty}-n$ for every $\widehat{0}+n\in\mathbb{N}$. The neighborhood of infinity is populated by the completion of $\mathbb{N}$ with the rationals and irrationals.

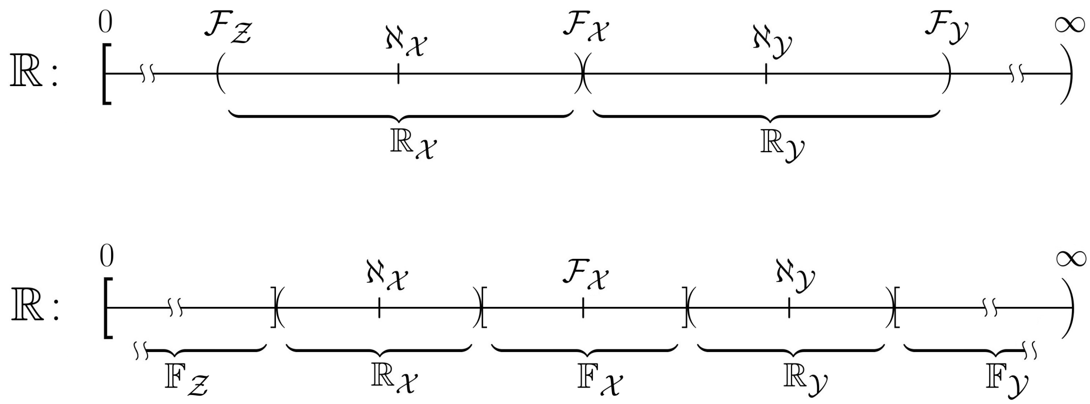

Figure 21: This figure demonstrates an open question in fractional distance analysis. Should the $\mathbb{R}_{\mathcal{X}}$ neighborhoods be separated by single non-arithmatic numbers $\mathcal{F}_{\mathcal{X}} \in \mathbb{F}$ (above) or should they be separated by entire neighborhoods of non-arithmatics labeled $\mathbb{F}_{\mathcal{X}}$ (below)?

between them, or if we might remove them from $\Sigma^{\pm}$ to colocate them with $\varnothing$ as their union: a single point separating $\Sigma^{\pm}$.

From $\mathcal{A}$ to Ω, the physical curvature of the slices of $\Sigma^{\pm}$ at constant $\chi^4_{\pm}$ is given by a monotonic function $R_{\pm}(\chi^4_{\pm})$.[1] R vanishes in the $\chi^4_{\pm} \to 0$ limit associated with $\mathcal{H}$, and it diverges or has divergent-like behavior at $\varnothing$. Although the there is a discontinuity at $\mathcal{H}$ where the scale changes, the Ricci scalar is continuous across $\mathcal{H}$. The topology changes from two timelike and three spacelike dimensions in Σ^- to one and four in Σ^+, but there is only a scale discontinuity in the geometry of the 4D slices.

At the $\Omega \to \mathcal{A}$ step of $\hat{M}^3$, the case is much different. In addition to a discontinuity in the topology and an implicit scale discontinuity, there is a stark jump discontinuity from Ω's non-vanishing positive geometric curvature to $\mathcal{A}$'s non-vanishing negative geometric curvature. While the discontinuity at $\mathcal{H}$ can be associated with the act of observation or the start and stop points for $\hat{M}^3$, the geometric and topological discontinuities at $\varnothing$ can only be associated with the reversal of the time arrow and the increased level of aleph. While it is not so hard to imagine the level of aleph changing the scale and metric signature, the asymmetric geometric discontinuity between positive curvature in Ω and negative in $\mathcal{A}$ is a harder problem. To solve such issues, it may be required to use the $\chi^A_{\varnothing}$ coordinates to resolve a space between Ω and $\mathcal{A}$. On the other hand, if we increase the magnitude of the curvature in Ω and $\mathcal{A}$

[1]The subscript on $R_{\pm}$ reflects a possible convention for increasing the curvature to $\pm\infty$ on $\mathcal{A}$ and Ω. This would require two different conformal infinity functions, i.e.: $R_+(\chi^4_+) = \tan(\varphi\pi\chi^4_+/2)$ and $R_-(\chi^4_-) = \tan(\Phi\pi\chi^4_-/2)$. However, this is not the case if we assume $R(\chi^4_{\pm}) = \chi^4_{\pm}$ such that the Ricci scalar is Φ on Ω and $-\varphi$ on $\mathcal{A}$. Also, this function will be written as $R_{\pm}(\chi^4_{\pm}, A^{\mu}_{\pm})$ most generally, but we ignore the second argument while $A^{\mu}_{\pm} = 0$.

to infinity, then the jump discontinuity from positive curvature to negative curvature is greatly simplified. For example, the problem of joining a singularity of infinite positive curvature to one of infinite negative curvature seems far simpler than joining two maximally symmetric spaces with finite Ricci scalars $R_1 \neq R_2$. Something as simple as a minus sign used to reverse the arrow of time might be used to swap infinite positive curvature for infinite negative curvature. Indeed, the physics of a black hole/white hole pair is exactly what is needed to continue a trajectory along χ^4 past the singularity at $\varnothing$. A state would fall into the black hole along χ^4_+ and then be ejected in reverse time along χ^4_-.

Proposals to separate Ω and $\mathcal{A}$ by an interval or to join them lead into the main problem in this section. Should Ω and $\mathcal{A}$ be branes of infinite curvature? Should they act like an event horizon surrounding $\varnothing$? If so, should an interval of $\chi^4_\varnothing$ separate $\chi^4_\pm$? Perhaps $\mathcal{A}$ and Ω should mark the onset of curvature in the neighborhood of infinity? Curvature in the neighborhood of infinity presents a case for new physics in fractional distance analysis because such curvature would describe a physical singularity but not a mathematical one.

The Ricci scalar in maximally symmetric $d+1$ dimensional Lorentzian spacetime[1] is

$$R_d = \frac{d\big(d-1\big)}{\pm\ell^2} \quad . \tag{4.1}$$

The de Sitter parameter ℓ defines dS or AdS as the induced metric on

$$-y_0^2 + y_1^2 + y_2^2 + y_3^2 \pm y_4^2 = \pm\ell^2 \quad , \tag{4.2}$$

where y_A are coordinates of flat 5D space satisfying

$$ds^2 = -dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2 \pm dy_4^2 \quad .^2 \tag{4.3}$$

(Even with $A^\mu_\pm = 0$, the 5D MCM metric $g^\pm_{AB} = \text{diag}(\eta_{\mu\mu}, \chi^4_\pm)$ is not flat in the fifth coordinate.) We have inserted the $\pm$ on ℓ^2 for concision in notation, but the radius of curvature in AdS is a number whose square is a negative number. That radius is timelike. Timelike or spacelike, the radius is called the de Sitter parameter

[1] Lorentzian topology is characterized by one sign different than others in the metric signature. Maximally symmetric Lorentzian spacetimes are Minkowski space, de Sitter space, and anti-de Sitter space. These correspond to $\mathcal{H}$ and the respective physical slices of $\Sigma^\pm$. Maximal symmetry indicates that the Ricci tensor is completely determined by the Ricci scalar.

[2] Allowing for imaginary distances, (4.2) is the equation of a 5D sphere. Together with (4.3), these equations make clear what is meant when it is said that dS and AdS are spheres of spacelike and timelike radii: AdS_4 is a Lorentzian sphere of timelike radius in a space of two timelike and three spacelike dimensions. The $\{-+++-\}$ metric signature in Σ^- specifies two timelike dimensions and three spacelike ones, so we have properly identified AdS_4 for the slices of Σ^-. Likewise, dS_4 is a Lorentzian sphere of spacelike radius in a space of one timelike and four spacelike dimensions, and it is fitting that the slices of the $\{-++++\}$ space Σ^+ are taken as dS_4.

and its square is inversely proportional to the Ricci scalar, as in (4.1). Minkowski space has $R=0$ which is implied by any de Sitter parameter in the neighborhood of infinity. To get finite R in the neighborhood of infinity, ℓ would have to be the square root of an infinitesimal number. In other words, the radius would have to be unphysical. However, the square roots of numbers with non-vanishing fractional distance with respect to infinity were treated in [2], Section 6.2 therein. The main result of that treatment was that no real number can be the square root of a number in the neighborhood of infinity, and the square root of an infinitesimal might follow a similar analytical program.

In addition to (4.1), the Ricci scalar is also defined as the contraction of the Ricci tensor. It is not immediately obvious what Ricci tensors might contract as scalars in the neighborhood of infinity, so these should be classified, and applications of the attendant metrics toward the region around $\varnothing$ must be studied. Even in the absence of such connections to the Ricci tensor, we might use the $\chi^4_\varnothing$ coordinate to continue $R(\chi^4_\pm)=\chi^4_\pm$ beyond $\mathcal{A}$ and Ω. This will allow an independent path for studying R in the neighborhood of infinity near $\varnothing$. In general, R being promoted to a function is a case of the generalized Brans-Dicke theory which will be briefly mentioned in Section 43. This is a natural framework in which to study curvature in the neighborhood of infinity.

If the Ω-brane marks the onset of curvature in the neighborhood of infinity, we must obtain

$$R_3(\Phi) = \mathcal{F}_0 \quad , \tag{4.4}$$

which is non-standard. If $\mathcal{F}_\mathcal{X} \in \mathbb{F}$ does not have arithmetic defined (which is why they are called non-arithmatic numbers), then how might a function of a real number have such an output? To answer this question, we should identify the $\mathbb{F}$ on one level of aleph with the $\mathbb{N}$ on a higher level, as in [2].[1] The Ω-brane marks the termination of Σ^+ at which point the level of aleph is expected to increase, so such a solution is well fitting. Overall, the $\varnothing$ region of the unit cell very much remains an unknown territory on our map.

5 Continuous Particle Creation and Annihilation

In the lab, it is often observed that one particle will decay to two particles. For instance,

$$\psi \rightarrow \chi + \phi \quad . \tag{5.1}$$

[1]The argument that the non-arithmatics on one level of aleph should be interpreted as the naturals on a higher level of aleph was presented in Remark 7.5.20 of [2].

However, there is no mechanism within the existing quantum theory by which the $\psi(x)$ function of one variable might smoothly evolve into the $\psi' = \chi(y) + \phi(z)$ function of two variables. Quantum theory gives us tools to determine an amplitude for a particle with k^μ to be found later as pair of particles with k'^μ and k''^μ, but the question of how we might get from here to there is not answered. Isham writes the following [68].

> "Consider a scattering experiment in which two particles collide and turn into three particles. Ignoring internal and spin quantum numbers, the initial and final states could be described by wavefunctions $\psi(\mathbf{x}_1, \mathbf{x}_2)$ and $\psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$. However, it is by no means obvious what type of time-dependent Schrödinger equation could allow a function of two variables to evolve smoothly into a function of three variables."

The $\widehat{\text{MCM}}$ operator was invented to affect the decay of the bounce state into two time arrow eigenstates [39]. The $U_\pm$ universes are charted in the separate coordinates $x^\mu_\pm$, but Isham points out that there does not exist an analytical equation to underpin and motivate

$$\widehat{\text{MCM}}\big|t_\star\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad .^1 \tag{5.2}$$

Fortunately, this is exactly the equation that we have now phrased as a change of basis operation between chronological and chirological time arrow states (Section 1.10.2). Since the $\psi \to \chi + \phi$ process in question deals with the *observation* of ψ and subsequently χ and ϕ, we are well motivated to invoke the physics of the MCM unit cell. The physics of time arrow basis states must be developed with the goal of solving this important and longstanding problem. Might we arbitrage a function of two variables from the change of basis and evolution operations on a function of one variable?

The following speculative mechanism explains how one might achieve the required dynamical increase of a function's variables. Beginning with a chronological time arrow eigenstate ψ in $\mathcal{H}_0$, one would represent it as a superposition in the chirological basis. Since Schrödinger evolution is a simultaneous process with chirological evolution, we might indicate non-decay with the superposition of chirological states being converted back to chronological states at t'. Decay would be indicated by the conversion of the states in the superposition back to the chronological basis at times t' and t'' respectively, such that the resultant expression is a function of two different spatial variables in $\mathcal{H}_1$. From $\psi(x)$ to $\chi(y) + \phi(z)$, y is obtained from the conversion of a chirological state at t' and z is obtained from conversion of another chirological

[1] One would bestow this universal equality with the causality inherent to decay by the incorporation of Heaviside functions. Such functions are said to impose *time ordering*.

state at t''.

6 Wavepackets Extending to Infinity

A standing problem in physics is the infinite spatial extent of wavepackets. We would like to construct analytical wavepackets localized in space, but this is not possible with standard mathematical tools. One way to summarize this problem is that the exponential function has no zeros on the real line, so wavepackets constructed from such functions must extend to $\pm\infty$. However, one of the main results to come of fractional distance analysis is that e^x actually does have an infinite number of zeros in $\mathbb{R}$:

$$x \in \widehat{\mathbb{R}} \quad \Longrightarrow \quad e^{-x} = 0 \quad . \tag{6.1}$$ [1]

This result should be extended to define a new framework for the analysis of wavepackets. Hypothetically, one would use the big parts of real numbers to model the lab scale across some spectrum of fractional distance while the wavepackets themselves would be defined with the small parts of real numbers so as to vanish outside of their local (comoving) neighborhoods.[2]

7 Dark Energy

Dark energy refers to a cosmological redshift of deep space supernovae consistent with an increasingly accelerating rate of cosmological expansion [33–35, 108]. However, increasing expansion is not consistent with any standard cosmological model. In standard cosmology, energy constraints are such that $E > 0$ causes the universe to expand forever, albeit under a decreasing rate of expansion. If $E = 0$, the universe will asymptotically stop expanding but never collapse. If the energy is negative (and the arrow of time points in the positive t direction), expansion will eventually stop, and the universe will recondense to a big crunch singularity. As a result, a theoretical energy called "dark energy" is supplemented to account for the additional expansion seen in the night sky. In this section, the main problem is to fit empirical dark energy survey data to a model in which the anomalous optical effect results from an interaction between two universes on opposite sides of a big bang-like singularity. This curve fitting exercise may be undertaken immediately without further preliminary inquiry. Aside from that, this section contains a discussion of the MCM framework

[1] Recall that $\widehat{\mathbb{R}}$ is the union of the maximal neighborhood of infinity with every intermediate neighborhood of infinity: $\widehat{\mathbb{R}} = \mathbb{R}_1 \cup \{\mathbb{R}_\mathcal{X}\}$ for $\mathcal{X} \in (0,1)$, as in Section 1.6.1.

[2] Recall that $\text{Big}(\aleph_\mathcal{X} + b) = \aleph_\mathcal{X}$ and $\text{Lit}(\aleph_\mathcal{X} + b) = b$ (Section 1.6.1).

for dark energy, and related investigations are indicated.

There are two pictures in which the MCM solution to dark energy may be set. The original solution [31, 39] is a simplified picture of Newtonian gravitation on cosmological scales while the second picture is framed in GR [109]. The first picture avoids the issue of anomalous spatial expansion in a cyclic cosmology model. Periodicity is imposed along x^0 such that each big bang is the aftermath of a big crunch at the end of a previous cycle of cosmology.[1] Such events are called *big bounces*. On the far side of the bounce at the end of our present cycle lies another universe with gravitational mass. If there exists a timelike interval unbounded in the future so that we might continue to measure proper time through the collapse of all of spacetime in a big crunch at t_1, then every $t > t_1$ labels a hypersurface of constant proper time having constant mass-energy M. Call the time of the next following bounce t_2, and let t' be the midpoint of the interval (t_1, t_2) between two bounces. In a certain simplification, Gauss' law allows us to consider Newtonian gravitation across the bounce between our current hypersurface of constant proper time t_0 and the integrated mass $\bar{M}$ of an M at every $t \in (t_1, t_2)$ as if $\bar{M}$ was a point mass located at t'. Figure 22 shows a Newtonian potential energy landscape in which our hypersurface of the present is treated as a test mass. The interaction is treated as 1D along the t axis because the bottleneck at the bounce should wash out any information about spatial distributions of matter-energy beyond the bounce. Due to the present being deeper into the gravitational well of $\bar{M}$ than supernovae on the past light cone, those images will appear to recede under acceleration in the rest frame of observers at t_0. This recession should be identical to dark energy.

Since it is not clear what "time integrated mass" is or what would be the mass-per-time integrated density, we might consider the same potential energy landscape in Figure 22 between the singularity at t_1 and observers at $(\vec{x}_0, t_0)$ rather than between $\bar{M}$ at t' and observers' entire slice of constant proper time. In that case, the radial nature of the interaction will be preserved, and dark energy will continue to depend only on Δt. We might also appeal to infinite relative scale across levels of aleph to establish observers in the present as small test masses gravitating with a larger mass in the future *and* on a higher level. However, the original mechanism in [39] described interaction across a bounce, Figure 22 essentially, and we have introduced

[1] The χ^4 periodicity across the unit cell is based in part on cyclic cosmology models such as loop quantum cosmology (LQC). Bojowald, the effective owner of the LQC theory, declared it dead in a 2013 talk. The record of this talk titled "Loop Quantum Cosmology: A Eulogy" [110] appears to have been deleted from the internet following a citation in [111] (excepted here as Appendix C.) It is not clear if LQC died on its merits or if it died for its association with a pseudo-plagiarism scandal that concluded with Jerry Sandusky's 2011 child rape indictment (Appendix C). However, the MCM is not attached to the specifics of the LQC model. Even the requirement for a cyclic cosmology in any form has been called into question because interaction along χ^4 may suffice to support the MCM mechanism for dark energy without an additional interaction across a big crunch in the distant chronological future.

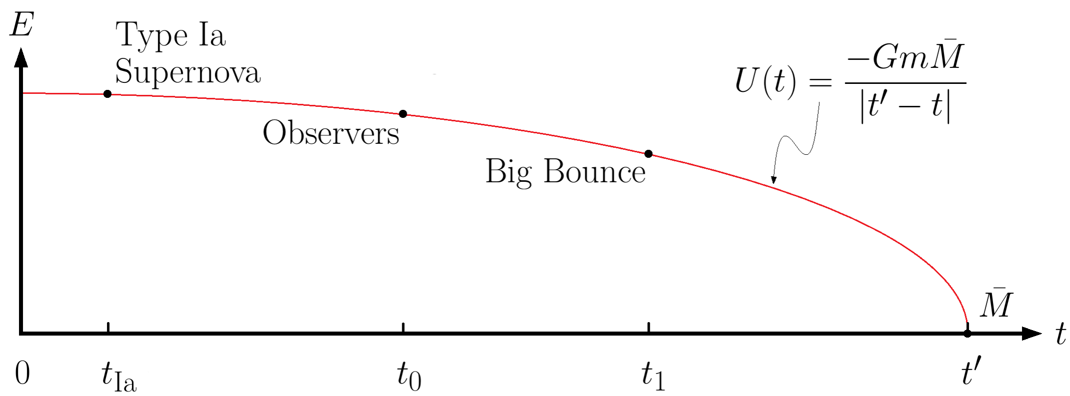

Figure 22: This figure depicts the Newtonian mechanism for MCM dark energy, mostly as it appeared in [31]. t_{Ia} is the proper time of a high redshift supernova in the dark energy survey. Such supernovae live on the past light cone of an observer with proper time t_0. The upcoming bounce lies at t_1. t' is the temporal midpoint of the cycle of cosmology beyond the bounce and $\bar{M}$ is a point mass representing the universe in the cycle beyond the bounce. The energy curve assumes that every slice of constant proper time in our present cycle of cosmology has the same mass m, so the energy between $\bar{M}$ and the slices of constant proper time at t_{Ia} or t_0 differs only in the timelike separation $|t'-t|$. Observers in an inertial frame at t_0 will see objects at t_{Ia} appear to recede under acceleration because t_0 is deeper into $\bar{M}$'s gravitational well.

the integrated mass so that we might treat the mass of a surface of constant proper time as a test mass in the well of the future cosmology cycle.

Unfortunately, we have avoided a question about the integrated mass in our current cycle of cosmology. Namely, if objects gravitate toward objects in the distant future, integrated or not, then they should also gravitate toward objects in the near future. For instance, a mass at time t experiencing gravitation with another m at $t+\Delta t$ would also gravitate with m at $t+\varepsilon$. Such effects regard what is called the gravitational backreaction or self-force. This is one of the most difficult subjects in GR, and in physics.[1] In an ordinary study of GR, geodesics describe the path followed by a test mass that does not disturb the metric as it moves. Parameterized motion along a predetermined geodesic does not account for the disturbance in the spacetime background caused by the changing position of a larger mass. Testifying to the difficulty of the problem, the full equations of motion by which a mass disturbs its

[1] The electromagnetic backreaction is given by the $\dddot{x}$ term in the Abraham–Lorentz force $F_{\mathrm{AL}} = m(\ddot{x} - \tau\,\dddot{x})$, so a third derivative may be implied in the corresponding gravitational backreaction. This implied derivative puts that effect squarely within the purview of the MCM, even before its present context in the solution for dark energy.

local curvature as it moves (self-gravitates) were not found until 1997 [112, 113]. If dark energy is to be an interaction between a mass in the present and the same mass in the future, then the MCM solution is a long distance backreaction. The interaction is self-gravitation by definition.

The Newtonian gravitational potential energy is such that the r in

$$U(r) = \frac{-Gm_1m_2}{r} \quad , \tag{7.1}$$

is restricted to $x^i \subset x^\mu$ spatial separations, but we will assume for the purposes of defining a gateway into this problem that this expression holds for $r = x^0$. GR puts space and time on the same footing, so this hand-waving is not unreasonable. Given an energy landscape as in Figure 22, type Ia supernovae will be observed from an inertial frame at t_0 to recede under acceleration. Observers are deeper into the gravitational well of $\bar{M}$, a large mass in the future, than distant supernovae on the past light cone, so they will experience greater Newtonian gravitation toward it than the supernovae. In observers' inertial rest frame, the difference in acceleration will be observed as supernovae receding under acceleration which increases as $(t_0 - t_{\text{Ia}})$ increases. Adding a second energy well associated with an earlier cycle of cosmology would compound the effect. The mechanism of the Newtonian solution to dark energy is that time is rarefied as the $1/r^2$ Newtonian force between universes pulls more strongly on late times than early times [31]. An observer in the present will accelerate toward the future more quickly than supernovae far back on his past light cone. The optical manifestation of this condition should be identical to the one attributed to dark energy: accelerating redshift which increases with the temporal displacement of optical images. The expansion of space in conventional dark energy theories is replaced with expanding time. Work remains to adapt the present Newtonian description to the language of GR, but it is likely that the simple description will be sufficient for a first order fit to the empirical data.

The energy landscape in Figure 22 neglects to account for the blueshift of photons falling into a gravitational well. Furthermore, gravitational time dilation[1] is such that clocks tick slower at lower gravitational potential while it seems like faster time in the present would be associated with acceleration toward the future. To account for such effects, one would add to Figure 22 an energy curve associated with an $\bar{M}$ in the previous cycle of cosmology and set the cosmological scale for dark energy $|t_{\text{Ia}} - t_0|$

[1] Consider the Schwarzschild metric $d\tau^2 \propto (1 - r_{\text{S}}/R)dt^2$ where r_{S} is the Schwarzschild radius, R is the distance from the singularity, t is the time on a clock at infinity, and τ is the time on a clock at R. Presently, r_{S} is like $|t' - t_1|$, and R is like $|t' - t_0|$. Without a second energy well due to a previous cycle of cosmology, a clock at t_{Ia} effectively measures Schwarzschild t while an observer's clock measures τ which is necessarily slower than t.

to be such that emitters at t_{Ia} are at lower energy than observers at t_0. As t_0 will accelerate more quickly toward future-directed $\bar{M}$ than t_{Ia}, and likewise for t_{Ia} toward past-directed $\bar{M}$, the appearance of recession for observers at t_0 will not be disrupted by the amended energy landscape.

For the second theoretical mechanism for dark energy, we will use the metric in place of the loose Newtonian approximation. The second picture is like an interaction between $\mathcal{H}$-branes separated by $\varnothing$ rather than two universes separated by a big bounce. A non-gravitational interaction is required because we have set the gravitational interaction between labeled branes to zero in Section 1.6.3. This was done to avoid gravitational collapse of the cosmological lattice, either by physical distance in the neighborhood of infinity or $U_{\text{grav}} \neq U_{\text{grav}}(\chi^4)$. In Section 1.7.3, we have shown that the scale factor $\mathcal{C}$ between levels of aleph k and j changes the energy as

$$E_k = \mathcal{C}^2 E_j \quad . \tag{7.2}$$

This is well suited to dark energy as a non-gravitational interaction across $\varnothing$ rather than, or in addition to, interaction across a big bounce. The lower energy of redshifted photons can be used to determine a direction for increasing $\mathcal{C}$.

Preliminary metrical analysis predicts an effect like dark energy in the unit cell without requiring actual gravitation [109].[1] The Friedmann–Lemaître–Robertson–Walker (FLRW) line element

$$ds^2_{\text{FLRW}} = -dt^2 + a^2(t)\big(dx^2 + dy^2 + dz^2\big) \quad . \tag{7.3}$$

describes flat expanding space. $a(t)$ is called the scale factor, and flat space stays flat as it expands because $a(t)$ is not a function of x^i. Borde, Guth, and Vilenkin describe the relation between $a(t)$ and redshift as follows [56].

> "Consider a model in which the metric takes the form
>
> $$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \quad . \tag{7.4}$$
>
> [...] From the geodesic equation one finds that a null geodesic in the metric, with affine parameter λ, obeys the relation
>
> $$d\lambda \propto a(t)dt \quad . \tag{7.5}$$
>
> Alternatively, we can understand this equation by considering a physical

[1] After examining the metric, we will ask whether a gravitational-like metric must necessarily imply gravitation.

> wave propagating along the null geodesic. In the short wavelength limit the wave vector k^μ is tangential to the geodesic, and is related to the affine parameterization of the geodesic by $k^\mu \propto dx^\mu/d\lambda$. This allows us to write $d\lambda \propto dt/\omega$, where $\omega \equiv k^0$ is the physical frequency as measured by a comoving observer. In an expanding model the frequency is red-shifted as $\omega \propto 1/a(t)$, so we recover [*(7.5)*]."

Photons propagate along null geodesics. A redshifted photon has lower frequency, so $\omega \propto 1/a(t)$ requires an accelerating increase in $a(t)$ to accommodate the observed accelerating increase in redshift. The general solution for a in FLRW cosmology is obtained from Einstein's equation under certain assumptions of homogeneity and isotropy. The result is that $a(t)$ decreases with time: the opposite of what is determined from deep space supernovae data [33–35, 108].

The MCM metric is

$$g^{\pm}_{AB} = \begin{pmatrix} g^{\pm}_{\alpha\beta} + \chi^4_{\pm} A^{\pm}_{\alpha} A^{\pm}_{\beta} & \chi^4_{\pm} A^{\pm}_{\alpha} \\ \chi^4_{\pm} A^{\pm}_{\beta} & \chi^4_{\pm} \end{pmatrix} \quad , \tag{7.6}$$

from which we obtain an $A_\alpha = 0$ line element

$$ds^2_{\pm} = -\left(d\chi^0\right)^2 + \left(d\chi^1\right)^2 + \left(d\chi^2\right)^2 + \left(d\chi^3\right)^2 + \chi^4_{\pm}\left(d\chi^4\right)^2 \quad . \tag{7.7}$$

Comparing ds^2_{FLRW} and ds^2_{MCM}, $\chi^4_{\pm}$ is a scale factor for the χ^4 part of $g^{\pm}_{AB}$.[1] Alternating sign for the scale factor follows from the $\pm$ subscripting because $\chi^4_{\pm}$ are oppositely positive- and negative-definite in the unit cell. Since we have not obtained a separable scale factor as when $a(t) \neq a(x^i, t)$ is the scale factor for the x^i part of the FLRW metric (the MCM has a non-separable $a(\chi^4)$ scale factor acting on the χ^4 part), the behavior of the non-linear scale factor must be investigated. It must be verified that the sign in the scale factor indicates redshift rather than blueshift. Assuming redshift is indicated, $a(\chi^4_{\pm}) = \chi^4_{\pm}$ describes distance in the χ^4 direction increasing with increasing χ^4. This effect was implemented along x^0 with the Newtonian $\bar{M}$ energy landscape, and now χ^4 has replaced x^0. The effect by which Newtonian gravitation toward the future rarefied chronological time is replicated with the non-separable scale factor on $(d\chi^4)^2$ rarefying chirological time. Furthermore, the increasing scale for χ^4 agrees with the scale we have associated with increasing levels of aleph.

In principle, we have demonstrated that a dark energy effect like gravitation be-

[1] We have previously commented on the possibility for taking $g^{\pm}_{44} = \pm|\chi^4_{\pm}|^2$ to match the quadratic form of ϕ^2 in the KK metric. The present $ds^2 \propto a^2(t)$ context for the FLRW scale factor also suggests that $g^{\pm}_{44} = \pm|\chi^4_{\pm}|^2$ might be better than the $g^{\pm}_{44} = \chi^4_{\pm}$ that appears in (7.7).

tween universes may be derived from the MCM metric between $\mathcal{H}$-branes. We have described this effect with Newtonian gravity and as a non-gravitational metric effect, but a question is begged whether a metric like gravitation must imply gravitation itself. In other words, do the branes gravitate after all? If so, it will be required to develop a mitigation preventing collapse such as Pauli degeneracy pressure between fermionic branes, for example. Perhaps infinitely increasing scale in the chirological future would imply a freefall-like equilibrium condition of metastable, eternal collapse-in-progress due to gravitation.

In the Newtonian model, a calculation is required to demonstrate that the energy term $U_{\text{grav}}(x^0)$ rarefies rather than compacts x^0, or that the given effect produces redshift rather than blueshift. In other words, it must be verified which of expanding or contracting time should be associated with cosmological redshift. In the metrical model, a calculation is required to show that increasing scale along χ^4 induces redshift rather than blueshift. Any disagreements will be remedied with a sign change.

8 Vacuum Energy

In QM, $\hat{x}$ is the position operator. In QFT, x is an index marking the field oscillator $\hat{\varphi}(x)$. Quantum oscillators have a famous zero point energy:

$$E = \frac{\hbar\omega}{2} \quad . \tag{8.1}$$

Due to the infinite number of points x in any non-zero volume, the energy density of the vacuum must be infinite. Since it is differences in energy that matter for physics, this constant vacuum energy is usually ignored. The MCM suggests two possible methods for dealing with infinite vacuum energy. First, fractional distance analysis provides the $\widehat{\infty}$ object with which we can choose not to ignore vacuum energy and track transfinite energy differences with arithmetic axioms [2] such as

$$\big(\widehat{\infty} + a\big) - \big(\widehat{\infty} + b\big) = a - b \quad . \tag{8.2}$$

However, a mathematical framework for handling infinite energies is disappointing because any connection to GR would cause the vacuum to collapse to a singularity. A second possible method for dealing with divergent vacuum energy would be to disassemble the foundations of QFT and reconstruct a theory in which vacuum oscillators oscillate jointly into $\Sigma^{\pm}$: one oscillation mode with $E = \hbar\omega/2$ and another with $E = -\hbar\omega/2$. Exotic models might be developed in which unequal probabilities for time arrow fluctuations lead to a non-vanishing but finite vacuum energy.

9 Spin Angular Momentum

If we associate the arrow of time with the direction of the p^0 component of the 4-momentum, then propagation though the unit cell is such that the the direction of p^0 alternates between $\Sigma^\pm$ and $\mathcal{H}$. As linearly independent degrees of freedom, x^0 and $\chi^4_\pm$ cannot point in the same direction. Following the picture of a right turn in the unit cell (Figure 4, Section 1.2.4), the p^μ vector must rotate, and we may infer an angular momentum from $L=I\dot{\theta}$. One would attempt to associate the fundamental increment of spin $\hbar$ with the total increment of angular momentum in the unit cell. The anomalous fractional increment $\hbar/2$ would be assigned to the $\Sigma^\pm$ halves of the unit cell. Furthermore, it is known that torsion is required to conserve spin in GR, so one would attempt to correlate spin derived from the rotation of the momentum 4-vector with torsion in the $g^\pm_{AB}$ metrics.

10 Spinor Structure from Spacetime

The Pauli matrices are a representation of the quaternions with

$$1 \to \mathbb{1} \quad , \quad \mathbf{i} \to -i\sigma_1 \quad , \quad \mathbf{j} \to -i\sigma_2 \quad , \quad \mathbf{k} \to -i\sigma_3 \quad , \tag{10.1}$$

or

$$1 \to \mathbb{1} \quad , \quad \mathbf{i} \to i\sigma_3 \quad , \quad \mathbf{j} \to i\sigma_2 \quad , \quad \mathbf{k} \to i\sigma_1 \quad . \tag{10.2}$$

The problem described in this section seeks to associate the Pauli algebra with the structure of spacetime by replacing the imaginary number in the timelike part of the Minkowski metric with a quaternion:

$$x^0 = ict \quad \longrightarrow \quad x^0 = \mathbf{q}ct \quad , \quad \text{where} \qquad \mathbf{q} \in \{\mathbf{i},\mathbf{j},\mathbf{k}\} \quad . \tag{10.3}$$

Notation for a complex phase between x^0 and t was developed in Section 1.2.4, but here we will briefly remotivate the convention.

The Lorentzian signature of Minkowski space is often taken as equivalent to the form of the Minkowski metric

$$g_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad . \tag{10.4}$$

Once the metric is defined, the differential element is

$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu \quad . \tag{10.5}$$

However, in the underlying theory of differential geometry pioneered by Riemann, we have a differential line element ds as the fundamental descriptor of curvature on manifolds. For physics, the quantity ds^2 is more useful, but everything needed for the mathematical construction of 4D manifolds with Riemannian curvature is encoded on

$$ds = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 \quad , \quad \text{with} \qquad a_\mu \in \mathbb{R} \quad . \tag{10.6}$$

To generate Lorentzian structure at this level, and specifically the Minkowski metric in (10.4), we set $x^0 = \gamma ct$ where γ has the property $\gamma^2 = -1$. Therefore, we may obtain the correct matrix representation of the $g_{\mu\nu}$ tensor if we use either of $x^0 = ict$ or $x^0 = \mathbf{q}ct$.

If we distinguish $\Sigma^\pm$ so that the Lorentzian structure of their respective dS and AdS slices are given by $\mathbf{q}_1$ and $\mathbf{q}_2 \neq \mathbf{q}_1$, the limit of small $\chi^4_\pm$ as $\Sigma^\pm$ approach a shared boundary at $\mathcal{H}$ is also the limit where the individual complex plane analogues will come into contact with a third embedding dimension. In other words, the quaternions in $\Sigma^\pm$ are only indistinguishable from the imaginary number until they come to the X-point at $\mathcal{H}$ (or $\varnothing$.) The algebra of the Pauli matrices is exactly the algebra of the quaternions, so one would seek to extract the Pauli algebra as a consequence of the metric in $\mathcal{H}$ being defined as a superposition of the $\chi^4_\pm \to 0$ limits in $\Sigma^\pm$. For example, given

$$x^0_+ = \mathbf{j}ct_+ \quad , \quad \text{and} \qquad x^0_- = \mathbf{k}ct_- \quad , \tag{10.7}$$

these two variables would not commute. They would anti-commute. Depending on the means by which physics in $\mathcal{H}$ is determined from the physics in $\Sigma^\pm$, one might obtain a Hamiltonian containing a product of such non-commuting variables. In the $\{+\,-\,-\,-\}$ Lorentzian signature convention calling for

$$\begin{aligned} x^1_+ &= \mathbf{j}x_+ & \qquad\qquad & & x^1_- &= \mathbf{k}x_- \\ x^2_+ &= \mathbf{j}y_+ & & \text{and} & x^2_- &= \mathbf{k}y_- \\ x^3_+ &= \mathbf{j}z_+ \quad , & & & x^3_- &= \mathbf{k}z_- \quad , \end{aligned} \tag{10.8}$$

the appearance of a product of non-commuting variables is especially easy to imagine when defining $\mathcal{H}$ through a matching condition on $\Sigma^\pm$ because the quaternions are attached to the spatial variables. In QM, a classical Hamiltonian $H = xp$ is quantized as

$$H = xp = \frac{1}{2}(xp + px) \quad \longrightarrow \quad \hat{H} = \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x}) \quad . \tag{10.9}$$

because neither of the $\hat{x}\hat{p}$ or $\hat{p}\hat{x}$ products can be favored. A similar lack of distinction between $x^i_+ x^i_-$ and $x^i_- x^i_+$ might be used in the present case to invoke the Pauli matrix commutator

$$[\sigma_j, \sigma_k] = 2i\varepsilon_{jkl}\sigma_l \quad . \tag{10.10}$$

In turn, this commutator algebra might drive the steering by right angles between successive $\Sigma^\pm$ which was prescribed earlier for avoiding metric signature discrepancies.

Fundamentally, integer QM spin states have classical counterparts while half-integer spin states do not. Since we have taken the $\mathcal{H}$-brane as the domain of quantum mechanics, we are well motivated to attempt to derive the half-integer spin algebra as the limiting algebra of the coming together of $\Sigma^\pm$ at $\mathcal{H}$.

11 Antisymmetry for Fermionic Wavefunctions

There is no theoretical motivation for the antisymmetry of fermionic wavefunctions. It is inserted into the framework of QM artificially to force agreement with experiment. Therefore, we should seek to motivate this antisymmetry from theory. The relative phase conventions for real, imaginary, complex, and oppositely signed $\chi^4_\pm$ seem well suited to such a development. The reversed time arrow between $\Sigma^\pm$, the association of $\hat{\imath}$ with $\varnothing$, the piecewise right turns of $\chi^4_\pm$ (Figure 4, Section 1.2.4), and the metric signature discrepancy between $\Sigma^\pm$ all provide leads which may have applications toward motivating fermionic asymmetry from first principles. The particle scheme in which fundamental fermions are constructed from single spacetime quanta while fundamental bosons are constructed from pairs may also have applications toward a theoretical motivation for symmetry and antisymmetry in bosons and fermions.

12 Time Arrow Spinors

In Section 1.10.2, MCM states were given as eigenstates of a time arrow operator: either the chronological $\hat{T}$ or the chirological $\hat{\mathcal{T}}$. The shared eigenvalue spectrum $\{+1, 0, -1\}$ indicated chronological $\{x^0_+, x^0, x^0_-\}$ or chirological $\{\chi^4_+, \chi^4_\varnothing, \chi^4_-\}$. If the time arrow operators are like an $\hat{S}_z$ operator, then we find a pair of spin-1 time states. In [84], however, time arrow spinors having spin-1/2 were developed as follows.

> "Through conservation of momentum we derive two times $t_\pm$ pointing in opposite directions from the big bang. We obtain the superposition time

$t_\star$ from $t_\pm$ via a quantum mechanical argument: the observer is unable to determine if he is in the universe with forward time or backward time. To an observer in either universe, time in that universe is forward, and the other is backward. This is in complete analogy with quantum mechanical spin: if the quantum observer cannot determine whether a two-state spin system is in the spin-up or spin-down eigenstate then he must write the state as a superposition

$$\big|\psi_\pm\big\rangle = c_\uparrow\big|\uparrow\big\rangle + c_\downarrow\big|\downarrow\big\rangle \quad . \tag{12.1}$$

Adapting from electron to universe, we write

$$\big|t_\star\big\rangle = c_+\big|t_+\big\rangle + c_-\big|t_-\big\rangle \quad . \tag{12.2}$$

with eigenspinors

$$\big|t_+\big\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad , \quad \text{and} \qquad \big|t_-\big\rangle = \begin{pmatrix}0\\1\end{pmatrix} \quad ." \tag{12.3}$$

The observer's inability to distinguish a positive time universe from a negative time one was the original motivation for defining time in the present as a superposition of positive and negative time:

$$\big|t_\star\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad . \tag{12.4}$$

This was also the main thinking in [39] when writing

$$\big|\text{bounce}\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad . \tag{12.5}$$

Both are in analogy with

$$\big|S_x;+\big\rangle = \frac{1}{\sqrt{2}}\big|\uparrow\big\rangle + \frac{1}{\sqrt{2}}\big|\downarrow\big\rangle \quad , \quad \text{or} \qquad \big|S_x;-\big\rangle = -\frac{1}{\sqrt{2}}\big|\uparrow\big\rangle + \frac{1}{\sqrt{2}}\big|\downarrow\big\rangle \quad , \tag{12.6}$$

meaning that spin-up and spin-down in the $\hat{x}$ direction may be expressed as independent linear combinations of $\hat{S}_z$ eigenstates. The reason for identifying $|t_\star\rangle$ with $|\text{bounce}\rangle$ was that the bounce should be the state of the present when the bounce happens. Subsequently, these states have been disassociated as chronological and chirological states. To motivate the disassociation on the same grounds as the former association, one would identify $|\text{bounce}\rangle$ with $t=\infty$ such that an observer's proper time in $|t_\star\rangle$ could never be that time. Presently, the spin-1/2 basis is not such that there should exist an eigenstate having eigenvalue 0. Time arrow eigenspinors return ±1 to their respective time operators, so clarification is required whether or not a complete time arrow basis has two states in it, or three. In Section 13, we will describe an MCM-specific supersymmetry rotation which might help resolve whether time states ought to be fermions or bosons, if a simpler resolution cannot be obtained.

The framework for bosonic MCM cosmology states in Section 1.10.2 must be reconciled with the spin-1/2 time arrow spinor states that are the topic of this section. The new mechanism for quantum gravity in Section 1.10.3 does not require that there are three states in the completeness relation since we have ignored one by choosing $c_\star = c_\varnothing = 0$. Neither do (12.4) or (12.5) require a third state with a zero eigenvalue. Indeed, the representations

$$\big|x^0\big\rangle = \big|x^0_+\big\rangle + \big|x^0_-\big\rangle \quad , \quad \text{and} \qquad \big|\chi^4_\varnothing\big\rangle = \big|\chi^4_+\big\rangle + \big|\chi^4_-\big\rangle \quad , \tag{12.7}$$

show that $|x^0\rangle$ and $|\chi^4\rangle$ cannot be eigenstates of the $\hat{T}$ and $\hat{\mathcal{T}}$ operators if

$$\hat{T}\big|x^0_\pm\big\rangle = \pm\big|x^0_\pm\big\rangle \quad , \quad \text{and} \qquad \hat{\mathcal{T}}\big|\chi^4_\pm\big\rangle = \pm\big|\chi^4_\pm\big\rangle \quad . \tag{12.8}$$

Any operator's eigenstates must be orthogonal, so one can never be expressed as a linear combination of the others. On the other hand, the expressions

$$\big|x^0\big\rangle = \big|\chi^4_+\big\rangle + \big|\chi^4_-\big\rangle \quad , \quad \text{and} \qquad \big|\chi^4_\varnothing\big\rangle = \big|x^0_+\big\rangle + \big|x^0_-\big\rangle \quad , \tag{12.9}$$

say that x^0 is a superposition of the adjacent $\chi^4_\pm$ times while $\chi^4_\varnothing$ is a superposition of the adjacent $x^0_\pm$ times. These expressions do not preclude the existence of a zero eigenvalue for $\hat{T}$ or $\hat{\mathcal{T}}$, and each forms a valid representation of $|t_\star\rangle = |t_+\rangle + |t_-\rangle$.

As an additional cog in the works, consider a $c_\star$ term added to the right side of

$$\widehat{\text{MCM}}\big|\text{bounce}\big\rangle = c_+\big|t_+\big\rangle + c_-\big|t_-\big\rangle \quad , \tag{12.10}$$

so that we obtain

$$\widehat{\text{MCM}}\big|\text{bounce}\big\rangle = c_+\big|t_+\big\rangle + c_-\big|t_-\big\rangle + c_\star\big|t_\star\big\rangle \quad , \tag{12.11}$$

which was (1.10.26) in Section 1.10.2. If $|t_\star\rangle$ is combination of $|t_\pm\rangle$, (12.11) reduces to (12.10) times a constant. We will have obtained obtain nothing new, and a zero eigenvalue is not suggested. Therefore, work is required to sufficiently parse the desired time arrow physics in terms of the chronological and chirological time states.

Following the structure of ordinary spin-1/2 states, we might introduce three time arrow operators $\hat{T}_{+}$, $\hat{T}_{0}$, and $\hat{T}_{-}$ corresponding to measurements of time-up or time-down in the $\{x^0_+, x^0, x^0_-\}$ directions. These would mimic the $\hat{S}_i$ operators as

$$\begin{aligned} \hat{T}_{+}\big|\psi;\hat{\Phi}\big\rangle &= \pm\big|\psi;\hat{\Phi}\big\rangle \\ \hat{T}_{0}\big|\psi;\hat{\pi}\big\rangle &= \pm\big|\psi;\hat{\pi}\big\rangle \\ \hat{T}_{-}\big|\psi;\hat{2}\big\rangle &= \pm\big|\psi;\hat{2}\big\rangle \quad , \end{aligned} \tag{12.12}$$

and we would introduce a similar algebra for $\hat{\mathcal{T}}_i$. Following an analogy with $\hat{S}_i$, (12.12) begs the question of a $\hat{T}^2$ operator commuting with the $\hat{T}_i$ such that

$$[\hat{T}_i, \hat{T}_j] = \varepsilon_{ijk}\gamma\hat{T}_k \quad , \quad \text{and} \qquad [\hat{T}_i, \hat{T}^2] = 0 \quad . \tag{12.13}$$

If two observable operators commute, the eigenstates of those operators should be described by two quantum numbers. Perhaps the time arrow operators supposed in Section 1.10.2 represent respective commuting observables so that simultaneous eigenstates might have two quantum numbers specifying an arrow of time (with a given scale) and time-up or time-down with respect to that arrow. Overall, the language for time arrow eigenstates is one of the most promising problems presented in this paper due to its high potential for immediate productive work.

Time arrow spinor states are well developed enough that we have obtained a Hamiltonian rather than only supposing that one should exist. The MCM Hamiltonian is based on the physics of the Stern–Gerlach experiment that separates spin superpositions according to their eigenstates, or combines them in its elaborate variants [84]. It was presumed that spinor-valued time states propagating in the MCM lattice, described as a **time circuit** in [84], would similarly separate and recombine as part of a general milieu. Spin-1/2 angular momentum states in the Stern–Gerlach experiment obey the Pauli equation

$$i\hbar\frac{\partial}{\partial t}\big|\psi_\pm\big\rangle = \left\{\frac{1}{2m}\left[\big(\hat{\mathbf{p}} - q\mathbf{A}\big)^2 - q\hbar\boldsymbol{\sigma}\cdot\mathbf{B}\right] + qA_0\right\}\big|\psi_\pm\big\rangle \quad , \tag{12.14}$$

where

$$A_\mu = (A_0, \mathbf{A}) \quad , \quad \mathbf{B} = \nabla\times\mathbf{A} \quad , \quad \text{and} \qquad \hat{H}_{\text{SG}} = -\frac{q\hbar}{2m}\boldsymbol{\sigma}\cdot\mathbf{B} \quad . \tag{12.15}$$

The $\hat{H}_{\text{SG}}$ part was adapted to $\hat{H}_{\text{MCM}}$ for time arrow spinors in the lattice as

$$i\hbar\frac{\partial}{\partial\chi^4}|\psi_\pm\rangle = \big(\hat{H}\underbrace{-ip^0\boldsymbol{u}_1\cdot\phi\mathbf{u}_2}_{\hat{H}_{\text{MCM}}}\big)|\psi_\pm\rangle \quad , \tag{12.16}$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are two quaternions and ϕ is the scalar field in the fifth diagonal position of the KK metric which we have identified with χ^4. $\hat{H}_{\text{MCM}}$ has replaced $\hat{H}_{\text{SG}}$, and we have condensed the remainder of the Hamiltonian down to $\hat{H}$ to avoid a question about the analogue of the kinematical momentum.

The steps to obtain this equation for $\hat{H}_{\text{MCM}}$ acting on time arrow spinors were as follows. The $|\psi_\pm\rangle$ momentum spinor was replaced with the $|t_\pm\rangle$ time spinor. We have rewritten the Pauli matrices with quaternions, as in Section 10. The substitutions

$$1 \to \mathbb{1} \quad , \quad \mathbf{i} \to -i\sigma_1 \quad , \quad \mathbf{j} \to -i\sigma_2 \quad , \quad \mathbf{k} \to -i\sigma_3 \quad , \tag{12.17}$$

allow us to write

$$\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k} \quad \longrightarrow \quad \boldsymbol{\sigma} = i\mathbf{u} \quad . \tag{12.18}$$

We have replaced the electric charge q with the energy p^0. Opposite energy in the two universes mimics opposite charge in the Stern–Gerlach apparatus. The energy of a complete universe is taken as one quantum in line with the electron's single quantum of charge. Following the canonical prescription, p^0 quantizes as $\hat{p}^0 = -i\partial_0$ which is the time derivative in Schrödinger's equation (up to a sign likely associated with metric signature.)[1] To avoid this double use, we have changed the time derivative on the left of (12.16) to ∂_4, but we might have alternatively replaced q with p^4 which would act on ϕ through $\phi^2 = \chi^4$. These cases for p^0 and p^4 should be complementary MCM Schrödinger equations, as was discussed extensively in [84]. Finally, the magnetic field $\mathbf{B}$ is replaced with the KK scalar field upgraded to a vector-like quantity by multiplication with $\mathbf{u}_2$. (The case of $\mathbf{B} \to \phi^2\mathbf{u}$ was also considered in [84].)

The MCM Hamiltonian and its physics require further study. In the case where the quaternions in $\hat{H}_{\text{MCM}}$ acquire non-unit magnitudes as in the ontological basis, first analysis in [84] shows that the expected energy ratios $E^-_{\text{MCM}}/E^+_{\text{MCM}}$ are remarkably like the $E_{\text{NRR}}/E_{\text{RR}}$ ratios observed in the negative frequency experiments of Rubino et al. [42, 44]. The latter ratio is the energy of negative resonant photons divided by that of resonant photons.

To the knowledge of this writer, exciting new algebraic structures developed in [84] have not appeared elsewhere in the literature. They are obtained by extension of

[1] Note that quantization of the p^0 component of the 4-momentum as $\hat{p}^0 = i\hbar c\partial_t$ allows us to write the time-dependent Schrödinger equation as $\hat{p}^0|\psi\rangle = c\hat{H}|\psi\rangle$.

the $x^0 = \mathbf{u}ct$ protocol for replacing the imaginary number with quaternions (Section 10). We will replace the imaginary number in the plane wave complex exponential as $e^{i\omega t} \to e^{\mathbf{u}\omega t}$. Then we will bring those waves into spinors and increase matrix complexity by converting the quaternions to nested Pauli matrices. Given a two component spinor wave

$$\big|\psi\big\rangle = e^{ikx} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad , \tag{12.19}$$

a quantum number is added to specify spin-up and -down as

$$\big|\psi;+\big\rangle = e^{ikx} \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} \quad , \quad \text{and} \qquad \big|\psi;-\big\rangle = e^{ikx} \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix} \quad . \tag{12.20}$$

For time spinors, we will use ψ and ξ to specify chronological and chirological eigenstates:

$$\big|\psi\big\rangle = e^{\mathbf{u}_1 kx} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad , \quad \text{and} \qquad \big|\xi\big\rangle = e^{\mathbf{u}_2 \beta\chi} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \quad . \tag{12.21}$$

To demonstrate what appears to be new matrix structure with an example, assume $\mathbf{u}_1 = \mathbf{j}$. Ignoring the ψ_1 part of $|\psi;+\rangle$ to use

$$\mathbf{j} \quad \longrightarrow \quad -i\sigma_2 = -i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \text{and} \qquad 1 \quad \longrightarrow \quad \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad , \tag{12.22}$$

we may write

$$\big|\psi;+\big\rangle = \begin{pmatrix} e^{\mathbf{j}kx} \\ 0 \end{pmatrix} = \begin{pmatrix} C + \mathbf{j}S \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbb{1}C - i\sigma_2 S \\ 0 \end{pmatrix} = \begin{pmatrix} C & -S \\ S & C \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{12.23}$$

This additional layer of matrix complexity is likely to support new channels for the flow of quantum information.

An additional system not described in [84] preserves the imaginary number in the plane wave exponent but allows the ontological labels to specify quaternion phase on time:

$$t \to t\mathbf{u} \qquad \Longrightarrow \qquad e^{i\omega t} \to e^{i\omega t\mathbf{u}}. \tag{12.24}$$

In this latter convention for complementing imaginary phase with quaternion phase, the program for finding the Pauli algebra in the structure of spacetime (Section 10) would revolve around $x^0 = ict\mathbf{u}$ rather than $x^0 = \mathbf{u}ct$. The identities

$$\sin(i\theta) = i\sinh(\theta) \quad , \quad \text{and} \qquad \cos(i\theta) = \cosh(\theta) \tag{12.25}$$

give another example of new matrix complexity as

$$e^{i\omega t\mathbf{j}} = \cos(i\omega t) + \mathbf{j}\sin(i\omega t) = \mathbb{1}Ch - i\sigma_2\big(iSh\big) = \begin{pmatrix} Ch & -iSh \\ iSh & Ch \end{pmatrix} \quad , \tag{12.26}$$

where Ch and Sh are $\cosh(\omega t)$ and $\sinh(\omega t)$. This can be further complicated with the imaginary number replaced as well. For example,

$$\begin{aligned} e^{\mathbf{i}(kx-\omega t\mathbf{j})} &= e^{\mathbf{i}kx}e^{-\omega t\mathbf{ij}} \\ &= e^{\mathbf{i}kx}e^{-\omega t\mathbf{k}} \\ &= \big(C(kx) + \mathbf{i}S(kx)\big)\big(C(\omega t) - \mathbf{k}S(\omega t)\big) \\ &= \big(\mathbb{1}C(kx) - i\sigma_1 S(kx)\big)\big(\mathbb{1}C(\omega t) + i\sigma_3 S(\omega t)\big) \\ &= \begin{pmatrix} C(kx) & -iS(kx) \\ -iS(kx) & C(kx) \end{pmatrix}\begin{pmatrix} iC(\omega t)S(\omega t) & 0 \\ 0 & -iC(\omega t)S(\omega t) \end{pmatrix} \\ &= \frac{1}{2}\sin(2\omega t)\begin{pmatrix} i\cos(kx) & -\sin(kx) \\ \sin(kx) & -i\cos(kx) \end{pmatrix} \quad . \end{aligned} \tag{12.27}$$

The utility of such structures toward new channels in quantum algebras must be evaluated.

13 Supersymmetry

The supersymmetric standard model of particle physics is well loved because the coupling constants of three of the four forces are unified at a certain energy (Figure 14, Section 1.9.4). A model is said to be supersymmetric if fundamental bosons have fermionic partners and fundamental fermions have bosonic partners. For each particle, one says there exists a supersymmetric *sparticle*. Therefore, we should consider the decompositions of chronological/chirological time arrow eigenstates as superpositions of chirological/chronological ones in the context of the MCM model of spin spaces (Section 1.4). This may help clarify a question about whether time arrow operators have bosonic eigenvalue spectra $\{+1, 0, -1\}$ or fermionic $\{+1, -1\}$.

Firstly, one considers the notion that if every time arrow eigenstate can be decomposed into a superposition of relative positive and negative time states, then one might construct a symmetric unit cell about whichever time eigenstate was decomposed. In essence, we might take any time as the time in an $\mathcal{H}$ analogue and treat the positive and negative parts of that time as $\chi^4_\pm$ analogues spanning $\Sigma^\pm$ analogues. Considering the case where we take a chirological time as the x^0 analogue, we would establish a system in which the equations for chronological time are like the equations for chirological time. Obviously, this mimics the usual supersymmetric notion of equality between equations for force and matter, or bosons and fermions. An integral concept in supersymmetry is the continuation of an ordinary Lie algebra onto what is called a super algebra, and that should provide guidance for the structure suggested here. Furthermore, if x^0_+ (for example) might be decomposed as some other variants of $\chi^4_\pm$ not present in the unit cell, then we establish a fractal model of infinite self-similarity.[1] Arkani-Hamed made a comment about how the spin-1 case for the Higgslike particle requires a "Russian doll" model of nested bosons [21], and the present suggestion is consistent with that analogy.

Consider a direct symmetry between bosons and fermions in the context of MCM spin spaces (1.4). Given the space of spin-1/2 states

$$L^2(\mathbb{R}^3)\otimes\mathbb{C}^2 \quad\equiv\quad L^2(\mathbb{R}^3)\otimes\chi^4_{+\{0\}}\otimes\chi^4_{-\{0\}} \quad , \tag{13.1}$$

the decompositions

$$\big|\chi^4_{+\{0\}}\big\rangle = c_1\big|x^0_{+\{0\}}\big\rangle + c_2\big|x^0_{\{0\}}\big\rangle \quad , \quad \text{and} \qquad \big|\chi^4_{-\{0\}}\big\rangle = c_3\big|x^0_{-\{0\}}\big\rangle + c_4\big|x^0_{\{0\}}\big\rangle \quad , \tag{13.2}$$

suggest

$$L^2(\mathbb{R}^3)\otimes\chi^4_{+\{0\}} \quad\longrightarrow\quad L^2(\mathbb{R}^3)\otimes\big(c_1x^0_{+\{0\}}\otimes c_2x^0_{\{0\}}\big)\otimes\big(c_3x^0_{-\{0\}}\otimes c_4x^0_{\{0\}}\big) \quad . \tag{13.3}$$

The equations in (13.2) decompose the $\chi^4_\pm$ in one unit cell in terms of their adjacent instances of chronological time. The result in (13.3) may be simplified by the $x\otimes x = x$ property of the tensor product as

$$L^2(\mathbb{R}^3)\otimes c_1x^0_{+\{0\}}\otimes c_5x^0_{\{0\}}\otimes c_3x^0_{-\{0\}} \quad\equiv\quad L^2(\mathbb{R}^3)\otimes\mathbb{C}^3 \quad . \tag{13.4}$$

This is the MCM supposition for the structure of the state space of spin-1 particles, as in Section 1.4. Therefore, one might derive a fundamental symmetry (a supersymmetry) from the underlying symmetry between representations of abstract quantum

[1] The structure by which each time arrow decomposition can be resolved on further time arrow decompositions has been said to make the MCM a fractal model of *infinite complexity*.

states in Hilbert space in the eigenbases of the chronological and chirological time arrow operators.

14 Representations of Quantum Algebras

We have introduced four new objects into quantum theory: the ontological basis vectors $\hat{e}_\mu$. Cases for the association of these objects and the their dyads with the four and sixteen generators of the Pauli and Clifford algebras should be explored. One would examine the utility of the ontological basis toward defining spinor/bispinor structure, e.g.:

$$\hat{\pi} \to \begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad , \quad \hat{\Phi} \to \begin{pmatrix} 0 \\ \Phi \\ 0 \\ 0 \end{pmatrix} \quad , \quad \hat{2} \to \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \end{pmatrix} \quad , \quad \hat{i} \to \begin{pmatrix} 0 \\ 0 \\ 0 \\ i \end{pmatrix} \quad . \tag{14.1}$$

As the Pauli matrices may be represented with quaternions, we would also explore cases for non-unit quaternion representations such as

$$\hat{\pi} \to \pi\mathbf{i} \quad , \quad \hat{\Phi} \to \Phi\mathbf{j} \quad , \quad \hat{2} \to 2\mathbf{k} \quad , \quad \hat{i} \to i\mathbb{1} \quad , \tag{14.2}$$

and similar. A convention for non-unit quaternions replacing Pauli matrices was important in [84] for matching the ratio of energies in $\Sigma^\pm$ to the energy ratio of the resonant and negative resonant photons observed by Rubino and McLenaghan et al. (Section 12) [42]. Additional sign conventions such as

$$\mathbf{u}_i\mathbf{u}_i = -1 \quad , \quad \text{and} \qquad \mathbf{u}_i\mathbf{u}_j = \varepsilon_{ijk}\mathbf{u}_k \quad , \tag{14.3}$$

may be useful for time arrow conventions. Quaternions (called $\mathbb{H}$) have the additional property

$$\mathbf{ijk} = -1 \quad , \tag{14.4}$$

which is not found in $\mathbb{C}$. However, a plane spanned by $\hat{1}$ and a unit quaternion $\hat{\mathbf{u}}$ must be exactly like $\mathbb{C}$ in the absence of at least a third embedding dimension to invoke any algebraic properties not inherent to $\mathbb{C}$.

$\mathbb{H}$ offers a geometric picture for thinking about tuples of complex numbers such as Pauli and Dirac spinors. The product $\mathbb{C} \otimes \mathbb{H}$ is a 5D space spanned by $\{\hat{i}, \hat{1}, \hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}\}$ containing what are essentially four complex planes.[1] A point in this 5D space is

[1] We take the overlap of the quaternion and complex identities as the identity, i.e.: $\hat{1} \otimes \hat{1} = \hat{1}$.

a tuple of four complex numbers when we associate $(1, \mathbf{u})$ with $\mathbb{C}$. Four complex numbers specify a Dirac spinor and the product $\mathbb{C} \otimes \mathbb{H}$ follows the suggestions for bestowing time arrow spinors with new matrix complexity in Section 12. $\mathbb{C} \otimes \mathbb{H}$ may be useful for moving the Dirac equation into the bulk of $\Sigma^{\pm}$. Associating the sixteen ontological dyads with sixteen distinct Dirac bra-kets in the form $\langle\psi;\hat{e}_\mu|\psi;\hat{e}_\nu\rangle$, one would seek to make associations with the Clifford algebra and its sixteen generators. One notes that associating each ontological basis vector with a Dirac matrix gives

$$\gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4 = \hat{2}\hat{\pi}\hat{i}\hat{\Phi} \quad , \tag{14.5}$$

which, in the limit of $\hat{\Phi}^k \rightarrow \hat{\Phi}^{\Delta k}$, associates the pervasive γ_5 matrix of the Dirac theory with the pervasive $2\pi i$ of complex analysis on a constant level of aleph. This feature and similar number-theoretical qualities derived from the association of the objects of standard quantum algebras with the ontological basis vectors should be investigated.

15 Mechanical Precession

Laithwaite has suggested that the anomalous precession of spinning discs [98] might be explained by the rate of change of the acceleration on the infinitesimal elements of the disc [97]. This context for the third derivative *directly* motivated the initial supposition that $\hat{M}^3$ should be a third derivative. In [3], we supposed that the apparent anti-gravity effects exhibited by spinning discs [98] might be attributed to a discrepancy between the time derivatives of the force in the past and future written as a power series in the fine structure constant. Although the EM interaction is unrelated, a potential relevance was inferred for the FSC because α should characterize the geometry of the unit cells exhibiting the assumed past/future discrepancy. We wrote the following [3].

> "The apparent anti-gravity effects witnessed in Laithwaite's gyro demonstration at the Royal Society *[98]* can be explained if there is a net force on $\mathcal{H}_i$ due to contributions from the past and future. Using [*the time derivative of centripetal force* $\dot{F} = mr\omega^3$,] we may write the following.

$$\begin{aligned} F_{net}\hat{\pi}_i &:= \sum_{n=1}^{\infty} \alpha^n \big(\dot{F}\,\hat{\pi}_{i+n} - \dot{F}\,\hat{\pi}_{i-n}\big) \\ &:= m\omega^3 \sum_{n=1}^{\infty} \alpha^n \Delta r_n \end{aligned} \tag{15.1}$$

> If this sum is taken to the continuum limit as an integral over time, the inclusion of the differential element dt will give the correct units. A 20 kilogram wheel was spun at 2500 revolutions per minute. Precession lifted the wheel 1.5 meters in 3 seconds. This created a constant linear $\hat{z}$-momentum. Dividing the impulse by the time we see the force of precession was about 3 newtons stronger than the gravitational force. Keeping terms to first order in α we derive a characteristic length scale for chiros.
>
> $$\begin{aligned} \vec{F}_p &= m\omega^3 \alpha\, \Delta r \hat{z} \\ 200 &= \big(20\big)\big(1.8 \times 10^7\big)\, \alpha\, \Delta r \\ \Delta r &\approx 10^{-4}\,\text{meters} \end{aligned} \tag{15.2}$$
>
> And that looks about right! Far from the nano-scale of quantum mechanics and far from the macro-scale of ordinary perception."

The extent to which 10^{-4} is a special number cannot be overstated. If we had obtained any n other than $-6 \leq n \leq -3$, the result could have been discounted immediately. Among the infinite possible integers, -4 is the most perfect one for new effects. The open question of new physics at this scale is discussed in [4, 5], for example. The calculation of this scale in [3] was the foundation for the arrangement of the unit cell being such that the metric in $\mathcal{H}$ should be obtained by the difference of the metrics in $\Sigma^{\pm}$, as in Section 0.2. The mechanism for metrical differences followed from the above calculation in which the disc's vertical rise is attributed to an asymmetry between contributions from the past and future. However, the problem of precession remains to be solved with formal equations of motion. Casting the motion of the disc as motion along a geodesic is likely to be productive because the expected vertical rise of the disc is already known.

16 The Advanced Electromagnetic Potential

This problem requires a breakdown and reanalysis of the foundations of the advanced and retarded potentials in Maxwell's equations. Eventually, the Maxwellian EM potential 4-vector must be defined in terms of the $A^{\mu}_{\pm}$ in the metrics of $\Sigma^{\pm}$:

$$g^{\pm}_{AB} = \begin{pmatrix} g^{\pm}_{\mu\nu} + f(\chi^4_{\pm}) A^{\pm}_{\mu} A^{\pm}_{\nu} & f(\chi^4_{\pm}) A^{\pm}_{\mu} \\ f(\chi^4_{\pm}) A^{\pm}_{\nu} & f(\chi^4_{\pm}) \end{pmatrix} \quad . \tag{16.1}$$

The following context for the advanced and retarded potentials appears in [114].

"In 1909 Walter Ritz and Albert Einstein (former classmates at the University of Zurich) debated the question of whether there is a fundamental temporal asymmetry in electrodynamics, and if so, whether Maxwell's equations (as they stand) can justify this asymmetry. As mentioned above, the potential field equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -4\pi\rho \quad , \tag{16.2}$$

is equally well solved with either of two functions

$$\phi_1 = \int \frac{\rho(x,y,z,t-r/c)}{r}\, dx\, dy\, dz \tag{16.3}$$
$$\phi_2 = \int \frac{\rho(x,y,z,t+r/c)}{r}\, dx\, dy\, dz \quad ,$$

where ϕ_1 is called the retarded potential and ϕ_2 the advanced potential. Ritz believed the exclusion of the advanced potentials represents a physically significant restriction on the set of possible phenomena, and yet it could not be justified in the context of Maxwell's equations. From this he concluded that Maxwell's equations were fundamentally flawed, and could not serve as the basis for a valid theory of electrodynamics. Ironically, Einstein too did not believe in Maxwell's equations, at least not when it came to the micro-structure of electromagnetic radiation, as he had written in his 1905 paper on what later came to be called photons. However, Ritz's concern was not related to quantum effects (which he rejected along with special relativity), it was purely classical, and in the classical context Einstein was not troubled by the exclusion of the advanced potentials. He countered Ritz's argument by pointing out (in his 1909 paper 'On the Present State of the Radiation Problem') that the range of solutions to the field equations is not reduced by restricting ourselves to the retarded potentials, because all the same overall force-interactions can be represented equally well in terms of advanced or retarded potentials (or some combinations of both). He wrote

> 'If ϕ_1 and ϕ_2 are [*retarded and advanced*] solutions of the [*potential field*] equation, then $\phi_3 = a_1\phi_1 + a_2\phi_2$ is also a solution if $a_1 + a_2 = 1$. But it is not true that the solution ϕ_3 is a more general solution than ϕ_1 and that one specializes the theory by

putting $a_1=1, a_2=0$. Putting $\phi=\phi_1$ amounts to calculating the electromagnetic effect at the point x, y, z from those motions and configurations of the electric quantities that took place prior to the instant t. Putting $\phi=\phi_2$ we are determining the above electromagnetic effects from the motions that take place after the instant t. In the first case the electric field is calculated from the totality of the processes producing it, and in the second case from the totality of the processes absorbing it. If the whole process occurs in a (finite) space bounded on all sides, then it can be represented in the form $\phi=\phi_1$ as well as in the form $\phi=\phi_2$. If we consider a field that is emitted from the finite into the infinite, we can naturally use only the form $\phi=\phi_1$, precisely because the totality of the absorbing processes is not taken into consideration. But here we are dealing with a misleading paradox of the infinite. Both kinds of representations can always be used, regardless of how distant the absorbing bodies are imagined to be. Thus one cannot conclude that the solution $\phi=\phi_1$ is more special than the solution $\phi=a_1\phi_1+a_2\phi_2$ where $a_1+a_2=1$.'

"Ritz objected to this, pointing out that there is a real observable asymmetry in the propagation of electromagnetic waves, because such waves invariably originate in small regions and expand into larger regions as time increases, whereas we never observe the opposite happening. Einstein replied that a spherical wave-shell converging on a point is possible in principle, it is just extremely improbable that a widely separate set of boundary conditions would be sufficiently coordinated to produce a coherent in-going wave. Essentially the problem is pushed back to one of ***asymmetric boundary conditions*** [*emphasis added*]."

Although the unit cell is depicted in a rectangular representation, the picture of Einstein's spherical wave converging on a point may be well suited to the MCM. The dark energy interaction described in Section 7 is radial about $\varnothing$, and it was supposed in Section 1.6.3 that higher levels of aleph might lie within $\varnothing$ rather than beyond it due to physical curvature in the neighborhood of infinity. Either arrangement is likely to support spherical wave shells in place of the plane waves we have considered for rectangular representations of the unit cell.

The MCM solution for classical electrogravity (Section 18) follows from an assumed condition

$$A^\mu\big|_{\mathcal{H}} = c_+ A^\mu_+\big|_\Omega + c_- A^\mu_-\big|_{\mathcal{A}} \quad , \tag{16.4}$$

where $A^\mu_\pm$ are like the ϕ_1, ϕ_2 in (16.3). The meaning of (16.4), which first appeared in [7], was that the usual A^μ in $\mathcal{H}$ is defined non-locally by $A^\mu_\pm$ on the $\mathcal{A}$- and Ω-branes. However, non-locality is unusual in EM. To that end, Zeh makes a concise statement in [115] regarding the physics which suggests a revision to (16.4).

> "Electromagnetic radiation will here be considered as an example for wave phenomena in general.[1] It may be described in terms of the four-potential A^μ, which in the Lorenz gauge obeys the wave equation
>
> $$-\partial^\nu\partial_\nu A^\mu(\mathbf{r},t) = 4\pi j^\mu(\mathbf{r},t) \quad , \qquad \text{with} \qquad \partial^\nu\partial_\nu = -\partial_t^2 + \Delta \quad ,^2 \tag{16.5}$$
>
> with $c=1$, where the notations $\partial_\mu := \partial/\partial x^\mu$ and $\partial^\mu := g^{\mu\nu}\partial_\nu$ are used together with Einstein's [*summation convention*]. When an appropriate boundary condition is imposed, one may write A^μ as a functional of the sources j^μ. For two well known boundary conditions one obtains the *retarded* and the *advanced* potentials,
>
> $$\begin{aligned} A^\mu_{\text{ret}} &= \int \frac{j^\mu(\mathbf{r}, t - |\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}' \\ A^\mu_{\text{adv}} &= \int \frac{j^\mu(\mathbf{r}, t + |\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}' \quad . \end{aligned} \tag{16.6}$$
>
> These two functionals of $j^\mu(\mathbf{r},t)$ are related to one another by a reversal of retardation time $|\mathbf{r}-\mathbf{r}'|$ [*sic*]. Their linear combinations are solutions of [*(16.5)*.]"

We may avoid an implication for non-locality in EM if we replace $A^\mu_+|_\Omega$ and $A^\mu_-|_{\mathcal{A}}$ in (16.4) with integrals as in (16.6). Therefore, we might write

$$A_\mu\big|_{\mathcal{H}} = c_+ A^+_\mu\big|_{\Sigma^+} + c_- A^-_\mu\big|_{\Sigma^-} \quad . \tag{16.7}$$

where $A^\mu_\pm|_{\Sigma^\pm}$ are integrated expressions. The second argument of j^μ would be adapted so that the given chronological retardation time describes chirological separation from $\mathcal{H}$. Considering only $\chi^4_\pm$, we would write

$$j^\mu(r, t \pm |r - r'|) \quad \longrightarrow \quad j^\mu_\pm(\chi^4_\pm, t \pm |\chi^4_\pm|) \quad . \tag{16.8}$$

[1] Although the Schrödinger equation is the heat equation rather than the intuitive wave equation, it falls under the topic of wave phenomena in general.

[2] Δ is the Laplacian operator: $\Delta \equiv \nabla^2 \equiv \nabla \cdot \nabla$. The formula for $\partial^\mu\partial_\nu$ reflects the convention $x^0 = ict$.

In (16.6), the same j^μ appears in both integrals, but we would derive separate $j^\mu_\pm$ from separate $A^\mu_\pm$ through the formula

$$\Box A^\mu_\pm = \frac{4\pi}{c} j^\mu_\pm \quad . \tag{16.9}$$

Among the many conventions for $\mathcal{H}$ and $\varnothing$ considered in previous sections, and for Ω and $\mathcal{A}$, we have not considered that Ω and $\mathcal{A}$ might be the limiting branes of $\Sigma^\pm$ around $\mathcal{H}$ so that the metric in $\mathcal{H}$ is defined as a sum (or difference) of the metrics in $\Omega, \mathcal{A}$. To compare this to the case where the metrics at $\chi^4_\pm \to 0$ contribute to $g_{\mu\nu}$ (Section 0.2), the lack of space between $\mathcal{H}$ and $\Omega, \mathcal{A}$ might be attributed to the scale of a certain level of aleph. In that case, (16.4) would not imply non-locality, the metric and the 4-potential in $\mathcal{H}$ would both be assembled from the $\mathcal{H}$-adjacent limits of $\Sigma^\pm$, we would obtain a chronological retardation time rather than a chirological one, and the suggested integrals over j^μ would refer to $x^i_\pm$ without $\mathbf{r}$ being extended to $\chi^4_\pm$. Therefore, this convention for abutting Ω and $\mathcal{A}$ should be added to the other ones proffered for future inquiry.

It should be noted how this convention highlights duality between $\mathcal{H}$ and $\varnothing$, or between $|t_\star\rangle$ and $|\text{bounce}\rangle$. We have asked many times whether Ω and $\mathcal{A}$ should be separated by an interval or a point, and we might say that they are separated by a point at $\mathcal{H}$ and an interval at $\varnothing$.

17 Kaluza–Klein Theory

A rigorous survey of Kaluza–Klein theory is required, e.g.: material covered in [8–10, 116–118]. Particularly, the MCM condition that physical branes are defined at constant values of the fifth coordinate has been supposed to generate KKT's cylinder condition in a natural way. This condition requires that 4D physics cannot depend on the fifth coordinate, but it remains to be shown rigorously that the MCM's braneworld scenario is consistent with the requirements. Furthermore, analysis is needed to separate Kaluza theory from Kaluza–Klein theory in which the fifth dimension is given a compact topology and small dimension. Methods of Fourier expansion in the fifth dimension to remove the compact topology condition must be studied and compared to the unit cell. Overall, KKT is rich and well documented, but the MCM has merely adopted its metric without making a full analysis. Such analyses must be carried out. For instance, the MCM should be justified independently under the Campbell–Magaard theorem[1] [119, 120] without appealing the case of that theorem

[1]The Campbell–Magaard theorem governs cases under which 4D solutions in GR may be embedded in flat 5D space.

implicit in KKT.

18 Classical Electrogravity

This problem regards the unification of electromagnetism and gravitation developed in [7]. The idea revolves around the definition of $\mathcal{H}$ as a sum of contributions from $\Sigma^{\pm}$. The $g_{\mu\nu}$ metric in $\mathcal{H}$ is taken as the sum (or difference) of the metrics in the 4D slices of $\Sigma^{\pm}$ at their respective $\chi^4_{\pm}\to 0$ limits, as in Section 0.2. In turn, the EM potential in $\mathcal{H}$ was to be taken as a sum of potentials in $\Sigma^{\pm}$. Thus, one might use an antenna to impose a certain A^μ in $\mathcal{H}$ which would define $A^\mu_{\pm}$ through the sum relationship. Since $A^\mu_{\pm}$ appear in the 5D metrics whose limits define $g_{\mu\nu}$, one should be able to steer $g_{\mu\nu}$ with an electrical antenna, or an array of them.

The sudden appearance of "hypersonic missiles" in late stage field testing in the years after the publication of [7] is taken as strong evidence that the MCM mechanism for electrogravity is sound. The lack of hypersonic missile technology approaching the late stage testing phase in the preceding years is explained by an important conceptual breakthrough related to electrogravity in [7].[1] Therefore, the work should be continued so as to obtain equations of motion by which the metric can be controlled with the EM potential. This problem is not expected to depend on $\hat{M}^3$ and should not amount to much more than crunching a large system of equations in a sufficient number of unknowns. This system of equations is laid out in [7], but a number of trivial deficiencies must be remedied before a physical, determinate system of equations can be presented.

19 ϕ^4 Quantum Field Theory

The MCM answers the fundamental problem of QFT with the spectrum of cosmological lattice modes (Section 0.3) [6], but the fundamental *applied* problem of QFT remains open. Namely, there is no analytical solution to

$$Z=\int D\phi\,\exp\left\{i\int d^4x\,\left\{\tfrac{1}{2}\left[\left(\partial\phi\right)^2-m^2\phi^2\right]-\tfrac{\lambda}{4!}\phi^4+J\phi\right\}\right\}\quad, \tag{19.1}$$

where $\phi=\phi(x^\mu)$, $J=J(x^\mu)$, and $D\phi$ is the Feynman path integral measure. It is called ϕ^4 theory due to the presence of the ϕ^4 term in Lagrangian needed to permit interactions between field excitations.

[1] Cook offers a fascinating account of classified electromagnetic propulsion technologies in [121]. A report of Alexander produced for NASA [122] contains related material which may be of further interest.

There are some mathematical problems, in the sense of rigor, with the notation for the infinite-dimensional integral over $D\phi$, but the present problem bears only on the integral in the exponential. Unresolved mathematical issues with $D\phi$ (Section 60) do not impede our ability to make predictions, but the lack of a general solution to the exponentiated integral is the major outstanding bottleneck on QFT's predictive capacity. In the absence of the anharmonic ϕ^4 term, the exponentiated integrand reduces to the Lagrangian density of the harmonic oscillator $\mathcal{L}_{\text{HO}}$ added to a source term $J\phi$. In that case, integration by parts yields a well known analytical solution. In the anharmonic case, the best we can do is the truncation of one or another infinite series to arbitrary order. The higher order terms become harder to calculate, and quantum field theorists would prefer an analytical solution in closed form. Since existing approximations for this integral rely on exponential series decompositions, one would examine cases for new methods reliant on the big exponential function E^x (Section 1.6.7) and new arithmetic axioms developed in fractional distance analysis [2].

Guralnik remarks on the ϕ^4 theory in [15].

> "The (Euclidean) action is given by:
>
> $$\int d^4x \left[\phi(x) \frac{\big(-\Box + m^2 \big)}{2} \phi(x) + g\frac{\phi^4(x)}{4} - J(x)\phi(x) \right] \quad . \tag{19.2}$$
>
> The (Euclidean) Schwinger Action principle:
>
> $$\delta \big\langle t_1 \big| t_2 \big\rangle = \big\langle t_1 \big| \delta S \big| t_2 \big\rangle \quad , \tag{19.3}$$
>
> results in the equation:
>
> $$\big(-\Box + m^2 \big)\phi(x) + g\phi^3(x) = J(x) \quad . \tag{19.4}$$
>
> Defining $\mathcal{Z}$ as the matrix element of a state of lowest energy in the presence of the source at very large positive time measured against the 'same state' at very large negative time and again using the Schwinger action principle leads to:
>
> $$\left[\big(-\Box + m^2 \big) \frac{\delta}{\delta J(x)} + g\left(\frac{\delta}{\delta J(x)} \right)^3 - J(x) \right] \mathcal{Z}(J) = 0 \quad . \tag{19.5}$$
>
> [...] A good way to make sense of this equation is examine it on a space time lattice with N space time points. This approach can be regarded as

the original definition of a quantum field theory which is realized only in the limit of vanishing lattice spacing. On a hyper-cubic lattice:

$$\Box\phi_n = \sum_k \left(\phi_{n+\hat{e}_k} + \phi_{n-\hat{e}_k} - 2\phi_n\right) \quad , \tag{19.6}$$

where $\hat{e}_\mu$ is a unit vector pointing along the k direction, and, for convenience, the lattice spacing is set to 1. Since functional derivatives become ordinary derivatives at a lattice point the equation for $\mathcal{Z}$ on the lattice is:

$$\left[-\sum_k \left(\phi_{n+\hat{e}_k} + \phi_{n-\hat{e}_k} - 2\phi_n\right) + m^2\frac{d}{dJ_n} + g\left(\frac{d}{dJ_n}\right)^3 - J_n\right]\mathcal{Z}[J_1, J_2, ...] = 0 \quad . \tag{19.7}$$

The space-time derivatives have served to make this an equation involving three lattice points with the functional derivatives becoming normal derivatives acting on the variable at the central lattice point."

This result suggests an appropriately structured variant of the discretized ϕ^4 problem using $\mathcal{A}, \mathcal{H}, \Omega$ as the three lattice points where the functional derivatives reduce to ordinary derivatives. Guralnik continues as follows [15].

"Zero space-time dimension means that only one point exists and thus the lattice equation becomes:

$$g\frac{d^3\mathcal{Z}}{dJ^3} + m^2\frac{d\mathcal{Z}}{dJ} = \mathcal{Z}J \quad . \tag{19.8}$$

While loosing any space-time structure and thus the possibility of understanding all the interesting structure that occurs in the continuum limit, the above still maintains the non-linear nature of quantum field theory and the associated multiple solutions. Calculating solutions is now straightforward. While the finite dimensional case potentially has an infinite number of solutions before accounting for the collapse of the solution set, the current equation, representing 'zero dimensional QFT' only has three independent solutions. The solutions can be found easily by using series methods."

Zero dimensional QFT is quantum mechanics. The reduction of the ϕ^4 QFT problem to that limiting case yields a third order equation similar to the MCM's expected equation for $\partial^3 + \partial$. Therefore, the easily obtained solutions referenced by Guralnik must be carefully studied.

20 Stimulated Emission from the Vacuum

This problem regards the anomalous amplification reported by Rubino et al. in "Soliton-Induced Relativistic-Scattering and Amplification" [44] which was a follow up to Rubino and McLenaghan et al.'s earlier paper "Negative Frequency Resonant Radiation" [42]. The following appeared in [44] (most citations removed.)

> "If compared to the well-developed field of traditional light scattering in which the medium is at rest, little attention has been devoted to the physics of scattering from a moving medium, in particular from a relativistically moving medium. Here we consider the remarkable ability of solitons to generate a co-propagating refractive index inhomogeneity that propagates at relativistic speeds. The basics of scattering from a time-changing boundary were discussed in detail by Mendonça and co-workers (see e.g. *[123]* and references therein). Examples of such 'time refraction' have been predicted and observed from a moving plasma front and in waveguide structures. Recently, the nonlinear Kerr effect, i.e. the local increase of the medium refractive index induced by an intense laser pulse, was proposed to induce a moving refractive index inhomogeneity within a dispersive medium such as an optical fibre. The laser pulse induced relativistic inhomogeneity (RI) was then described in terms of a flowing medium in which the analogue of an event horizon may form and applications such a optical transistors have been proposed. Intense laser pulses are also known to scatter from the self-induced travelling RI: this self-scattering process leads to the resonant transfer of energy from the laser pulse to a significantly blue-shifted peak, often referred to as resonant radiation (RR) or 'optical Cherenkov' radiation. A recent discovery highlighted an additional scattered mode, further blue-shifted with respect to the RR, identified as a mode excited on the negative frequency branch of the medium dispersion relation and therefore named 'negative resonant radiation' (NRR)."

Firstly, the present author's area is not experimental quantum optics. Secondly, this writer has undertaken only a cursory survey of the results in [42, 44]. That being said, with frequency being the inverse of time, and with the negative frequency result following so closely on the heels of the MCM result regarding negative time, it is suggested that NRR is an MCM corollary result. Effects cited by Rubino et al. including time refraction, the analogue of an event horizon, and self-induced traveling relativistic inhomogeneities seem to make qualitative allusions to:[1]

- dark energy as time refraction (described as time rarefication in Section 7),
- the event horizon surrounding $\varnothing$ between $\Sigma^{\pm}$, and less obviously
- to the $\mathcal{H}$-brane as a relativistic inhomogeneity disrupting the uniformity of $R_{AB} = 0$ in $\Sigma^{\pm}$.

Even the title of Mendonça's book, *Theory of Photon Acceleration* [123], makes a qualitative allusion to the boosted photons cited by Particle Data Group as making an allowance for the Higgslike particle to have spin-1 (Section 0.1) [27, 124, 125]. In general, the NRR result must be dissected and meticulously understood.

Before undertaking such a comprehensive study, an underlying mechanism has been supposed as the cause of the amplification reported in [44]. Lasers are monochromatic and in phase, so it is possible to create surfaces of phase lock within crossed laser beams. A surface of $|\mathbf{E}| = 0$ would necessarily be solitonic because it represents the absence of the $\mathbf{E}$ field. It would be a relativistic inhomogeneity in the crossed lasers due to the $|\mathbf{E}| = 0$ condition juxtaposed with the broader $|\mathbf{E}| \neq 0$ laser field. The velocity of the $|\mathbf{E}| = 0$ surface should be relativistic on the order of the beam group velocity. Vanishing $\mathbf{E}$ is the boundary condition defining the surface of a piece of metal in classical EM, so the surface of phase lock might act as a virtual 2D metal foil in the path of the beam. We conjecture that the anomalous amplification in the negative frequency mode is the analogue of the photoelectric effect on the virtual foil. As a 2D surface, it is equipped with two oppositely signed normal vectors, one which is available to cancel the minus sign associated with negative frequency so that physical negative resonant photons might be observed with positive energy in the lab frame. One would attempt to describe this process as **stimulated emission from the vacuum**: a second quantized version of stimulated atomic emission in first quantization. Supporting such a mechanism, Rubino et al. write the following in [44] and [42] respectively.

> "We have shown that a [*relativistic inhomogeneity*] amplifies and scatters light to higher frequencies. Likewise, if the probe pulse were to be reduced to the level of quantum fluctuations, we may expect to see the RI excite the vacuum states."

> "A process such as that highlighted here, that mixes positive and negative frequencies will therefore change the number of photons, leading to

[1] It is not suggested that the authors of [42, 44] allude to the MCM's elements. Rather, it is suggested that the physical context for negative resonant radiation overlaps with the MCM's physical context.

amplification or even particle creation from the quantum vacuum."

It must be investigated whether a theoretical mechanism for stimulated emission from the vacuum might be useful for describing such processes. If so, this should have direct application toward the construction of *overunity electrical devices* powered by vacuum energy.

21 The Dual Tangent Space

The dual tangent space was heavily emphasized in a previous MCM review [1]. The expected utility for this space is to facilitate smooth propagation from Σ^+ into Σ^- without being blocked by a singularity at $\varnothing$. As an example of a desirable mechanism, the interaction cross section between two large but oppositely signed momentum states is low even when the spatial overlap of the states is high. We might seek to develop a third representation beyond the position and momentum spaces in which the interaction cross section between a state transiting the unit cell and the singularity at $\varnothing$ is also low. One "goes into the tangent space" from position space by taking the Fourier transform to obtain a momentum space representation. The usual framework of the Fourier transform and its inverse do not suggest a third representation beyond position/momentum space, but one would seek to associate some dual tangent representation with the χ^4 direction. In a familiar way, the binormal vector is perpendicular to the tangent space, and we might introduce another Fourier-like transform to **abstract space** as a third case beyond position or momentum space. This case would be associated with the dual tangent space as momentum space is associated with the tangent space. While such descriptions are not usual in physics, the additional derivative in the expected ∂^3 operator suggests structure beyond what is usually derived from the $\dot{p}=m\ddot{x}$ relationship.

22 Reverse Time in Quantum Field Theory

The early goal in treating the universe as a quantum particle[1] was to resolve an important question left unanswered by the standard model of particle physics: why does matter dominate over anti-matter in the universe? This question is called the mystery of the matter asymmetry [63]. The MCM solution is that a momentum-conserving pair of universes $U^{\pm}$ are dominated by matter and anti-matter respectively so that there is no global excess. In essence, one universe is a particle, and the other is an anti-particle. The pair is said to come into existence at a fluctuation

[1]This refers to work in [31] which predates the MCM model of particles in [6] by about three years.

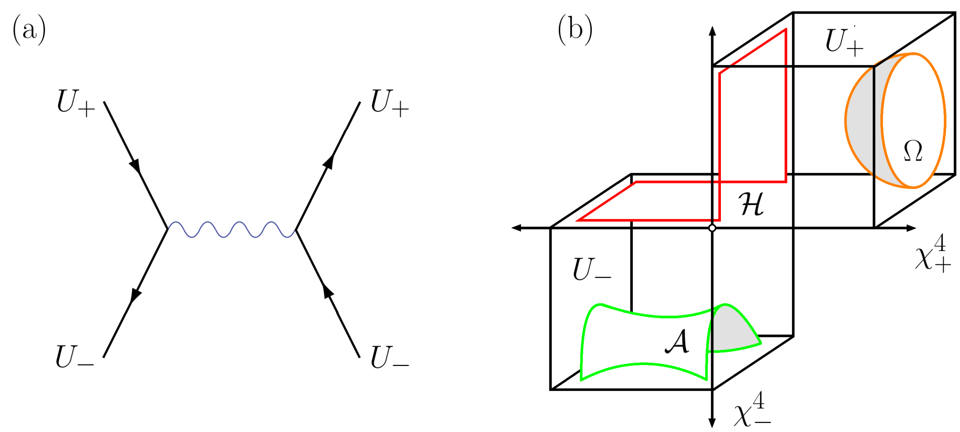

Figure 23: (a) A Feynman diagram for electron-positron annihilation is such that time increases to the right. It is labeled for comparison to the arrangement of time arrows in the unit cell. The reversed arrow on the U_- legs indicates that U_- is an anti-particle. If one pair of $U_\pm$ labels were swapped and time was to point in the vertical direction, this annihilation diagram would become a scattering diagram whose time arrows are still evocative of the structure of the unit cell. (b) This figure demonstrates that the restriction to positive- or negative-definiteness for $\chi^4_\pm \in \Sigma^\pm$ may be inherited from two unbounded intervals of $\chi^4_\pm$ which exceed what is contained within the unit cell. By inserting another instance of the unit cell into the second and fourth quadrants, one exactly replicates the structure of the Feynman diagram.

called a big bang or big bounce. The problem described in this section calls for an investigation to determine the extent to which QFT's interpretation for particles and anti-particles moving oppositely through time might be useful for describing MCM cosmology states.

The model of electrons and positrons interacting by coming together along opposite motions through time should be well suited to two oppositely timed universes $U^\pm$ coming together at a big bounce and then separating, as in Figure 23a. They annihilate to a photon-like bounce and then emerge from a null interval-analogue via a process like $\gamma \to e+p$. Ignoring the pre-bounce epoch, a pair of universes with opposite time arrows coming into existence is like the process for pair creation by vacuum fluctuations. The disjoint representation of the unit cell in Figure 23b enables an easy visualization of the particle scattering diagram as a cosmology process. For one transit of the unit cell, we have a universe coming into the null interval analogue, $\mathcal{H}$, from $\mathcal{A}$ and then going back out toward Ω. Since the arrow of time points oppositely in $\chi^4_\pm$, this process should be associated with the left or right side of the scattering diagram: half the path of U_+ and half that of U_-. The continuation of $\chi^4_\pm$ beyond their respective positive and negative subsets in $\Sigma^\pm$ (Figure 23b) describes the continued paths for $U_\pm$. A choice to represent the unit cell with $\Sigma^\pm$ joined on $\mathcal{H}$ or $\varnothing$ corresponds to another choice between assembling $U^\pm$ from one side of the Feynman diagram, or from opposite corners. In one case, $U_\pm$ are separated by nothing, and in the other they are separated by the null interval. Indeed, the Feynman diagram suggests that we might complete the likeness by squeezing a second instance of the unit cell into the second and fourth quadrants of Figure 23b. We have previously made implicit reference to these quadrants when discussing $(\hat{M}^3)^\dagger$ and $\hat{\varphi}$ in Sections 1.2.4 and 1.2.5.

The MCM bouncing mechanism and unit cell are strikingly like the most famous Feynman diagram. To move forward with this correspondence, it must be determined which QFT processes are best suited to the modified model of cosmology. Feynman diagrams represent amplitudes, but it is not immediately obvious in what way an amplitude might describe our present universe in progress. However, the AdS/CFT correspondence is famously exciting despite the absence of any direct utility for it. Such *correspondences* are exciting in physics because they are believed to be important.

23 Absorber Theory

The Wheeler–Feynman absorber theory of classical electrodynamics [126–128] supposed the physical existence of advanced and retarded solutions to Maxwell's equations. By doing so, they were able to accurately describe physics at almost all length scales under the assumption that particles do not self-interact. This is the opposite of the Abraham–Lorentz force in which the radiation damping term $\dddot{x}$ is a pure self-interaction. Despite the absorber theory's enormous successes, it was eventually rejected on its failure at small length scales. It was acknowledged that there is no good reason why an electron might not emit a photon and then later absorb that same photon. However, early thinking which did not pan out was the foundation for later methods in QED where particle self-interactions are of the utmost importance, *and* where the context for advanced and retarded times survives, as in Section 22. Considering that the absorber theory was supplanted by the Abraham–Lorentz law which is known to have several issues of its own and rely on a questionable period-averaging procedure, one would reexamine the fundamentals of the absorber model to determine whether or not new MCM physics might bridge the gap where it is said to have failed.

In the opinion of this writer, the *triple-C* cosmic censorship conjecture against the existence of naked singularities has euphemistic overtones referencing censorship in the advanced black hole physics literature. Therefore, one wonders if Feynman truly rejected the absorber theory which had similar interesting things to say about the nature of time. Perhaps censorship in the "national security" apparatus required that he disavow a valid theory? The present problem requires a full reevaluation of the absorber model which nearly worked, but is said to have ultimately failed. Rather than taking the word of those who have looked at it previously, the validity or invalidity of the theory must be independently certified.

24 Renormalization and Regularization

Methods of renormalization and regularization in QFT are pathological to the extent that they could be included in the MCM's list of targeted issues in quantum theory (Section 1.1.3). Both methods pertain to problematic infinities that arise when computing amplitudes, so both are well suited to reanalysis in the MCM and its fractional distance framework.

To sketch a path of investigation, the method of iterative updates to calculations-on-the-fly called *renormalization* is very much like what we have called translation of the observer's reference frame onto a higher of level of aleph. If we don't re-normalize to the scale of the higher level of aleph, we expect $\hat{M}^3$ to output numbers in the neighborhood of infinity that are not useful for comparison to physical quantities observed in $\mathcal{H}$-branes. Quantum theory will be much improved if unnatural techniques of renormalization are solidified in the context of changing scale from one level of aleph to another. *Regularization* in QFT, on the other hand, is introduced to avoid undefined quantities such as $\infty - \infty$, but such expressions are defined with $\widehat{\infty}$ in fractional distance analysis. Even the regularization cutoff scales imposed by *regulators* might be better implemented when the $\mathbb{R}_\mathcal{X}$ local neighborhoods of fractional distance provide inherent, sub-infinite cutoffs. An extensive survey of such methods should be conducted with the intention to regularize or normalize what are currently two irregular and abnormal methods in physics.

Testifying to the pathology of such methods, Dirac is quoted by Kragh as follows [129].

> "Most physicists are very satisfied with the situation. They say: 'Quantum electrodynamics is a good theory and we do not have to worry about it any more.' I must say that I am very dissatisfied with the situation because

> this so-called 'good theory' does involve neglecting infinities which appear in its equations, ignoring them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves disregarding a quantity when it is small—not neglecting it just because it is infinitely great and you do not want it!"

Similarly, Feynman wrote the following [86].

> "The shell game that we play is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate."

25 AdS/CFT Correspondence and Holographic Duality

A key new insight in the MCM is concisely expressed in terms of holographic duality. The solution to electrogravity (Section 18) [7] is a direct application of *holographic* bulk-boundary correspondence. This principle, broadly called AdS/CFT correspondence due to a famous context discovered by Maldacena [58], allows one to describe physics in a N dimensional bulk as a theory on a boundary surface in $N-1$ dimensions. The AdS/CFT correspondence is specifically such that physics in 5D anti-de Sitter space is determined by a conformal field theory (CFT) whose domain is only the boundary of AdS_5. The usual program in bulk-boundary correspondence is that the holographic surface is taken as the exterior of one bulk, but the new idea in the MCM is to put the holographic surface between two bulks: 4D $\mathcal{H}$ between 5D $\Sigma^\pm$. Therefore, one would make a survey of the primary applications of the extensively treated AdS/CFT problem posed by Maldacena as well as the broader contexts for holographic symmetries in physics. One would attempt to continue the duality of one bulk and one surface to two bulks and a surface, or two bulks and two surfaces ($\mathcal{H}$ and $\varnothing$) with the goal to make non-trivial advancements in understanding holographic correspondence and its (missing) use cases in physics.

26 Numerical Analysis

Among the 66 theses in this paper, this problem is likely to be the most productive. A multitude of unanswered questions about the exact analytical structure of MCM mechanisms will be brightly illuminated when the ideas are implemented as an exercise in the numerical analysis of arrays. The analysis of numerical solutions and their visualizations with computer graphics is likely to be more efficient and less cumbersome than a long slog through the analytical underpinning of everything (which will be necessary and delightful.)

A computational environment for simulating physics in the MCM unit cell must be developed. One would begin with numerically integrated wave equations in flat 5D spacetime. One would generalize to 5D wave equations in the $\{-+++\pm\}$ signatures and study the conditions for their smooth transmissiblities. The array structure of numerical analysis on grids is likely to provide immediate and keen insights regarding what is required for controlling transmission and reflection coefficients at the interfaces between $\Sigma^{\pm}$. One would graduate to simulations of waves in curved space [130, 131]. Having implemented the wave equation in curved spacetime, one would employ optimization to find the $R_{AB}=0$ bulk solutions allowed by KKT. One would explore gravitational waves and scalar EM waves, at least one of which is consistent with a vanishing Ricci tensor. One would make extensions to the heat equation. This long exercise in numerical analysis is likely to answer many open questions regarding the structure of the unit cell and the equation for $\hat{M}^3$. The visual representation of simulated time evolutions will be an invaluable aid.

27 The Location of the Observer

The usual space of wavefunctions in the position representation is L^2, the space of square integrable functions:

$$L^2(\mathbb{R}) \ni \psi : \mathbb{R} \to \mathbb{C} \qquad \Longrightarrow \qquad \int_{-\infty}^{\infty} dx \left|\psi(x)\right|^2 < \infty \quad . \tag{27.1}$$

The physics of the L^2 condition is that the wavefunction must support the probability interpretation. Being less than infinity, the integral of the absolute square of the wavefunction over all of space can be normalized to unity. This tells us that the probability of finding the particle somewhere in the universe is 100%. The L^2 condition also tells us that the probability of finding the particle at infinity is 0%. Sometimes, the L^2 condition is informally stated as a requirement that $\psi(\infty) = 0$. When one comes across a common phrase in physics, "Assuming boundary terms at infinity go to zero," this is a reference to the assumed $\psi(\infty) = 0$ condition, as in Section 28. The Cauchy residue theorem is often applied with part of a closed integration path at infinity where the integrand vanishes due to $\psi(\infty)=0$.

New boundary conditions are always a first thought in the search for new physics. Much of quantum theory is constructed around the $\psi(\infty)=0$ condition. However, there exists one other place where the probability of finding the particle vanishes, one that has been little considered, if at all: the location of the observer. If the observer is located at x_0, then $\psi(x_0)=0$, but this fact is not reflected in the usual approach to QM. As a matter of practice, the MCM convention is to place the observer at the origin, so one would construct a new state space of wavefunctions which go to zero at infinity *and at the origin.*

In addition to new boundary conditions, new symmetries are also highly regarded in the search for new physics. In fact, symmetries are a type of boundary condition. Using the one point compactification of $\mathbb{R}$,[1] namely $\mathbb{R}\cup\{\infty\}=\mathbb{S}^1$, the L^2 condition may be approximated as

$$\psi(x)\in L^2 \quad\implies\quad \begin{cases} \psi:\mathbb{S}^1\backslash\{\infty\}\to\mathbb{C} \\ \lim\limits_{x\to\infty}\psi(x)=0 \quad . \end{cases} \tag{27.2}$$

Calling the proposed subdomain which incorporates the position of the observer L_0^2, we may write

$$\psi(x)\in L_0^2 \quad\implies\quad \begin{cases} \psi:\mathbb{S}^1\backslash\mathbb{S}^0\to\mathbb{C} \\ \lim\limits_{x\to 0}\psi(x)=0 \\ \lim\limits_{x\to\infty}\psi(x)=0 \quad . \end{cases} \tag{27.3}$$

$\mathbb{S}^0$ is two points, and we have excluded $x=0$ and $x=\infty$ from the domain of functions in L_0^2. This represents a radical change in the topological structure of quantum theory, and it may provide powerful new tools for doing quantum mechanics. Furthermore, the Lorentzian structure of spacetime is such that we may treat $\psi(x,t)$ as if it were a function $\psi(z)$ of a single complex variable through $\hat{x}\to\hat{1}$ and $ic\hat{t}\to\hat{i}$, as in Sections 1.2.4 and 10. Denoting the Riemann sphere $\mathbb{S}^R=\mathbb{S}^2\backslash\{\infty\}$, we have

$$\psi(x,t):\mathbb{S}^R\to\mathbb{C} \quad\longrightarrow\quad \psi(x,t):\mathbb{S}^2\backslash\mathbb{S}^0\to\mathbb{C} \quad .^2 \tag{27.4}$$

The removal of the origin from the domain of ψ generates a new topology with more symmetry. The old domain was a sphere missing a point, a famously asymmetric object in analysis. The new domain is the topological difference of two spheres.

[1]The one point compactification imposes the circular topology on $\mathbb{R}$ by joining the unincluded endpoints of $(-\infty,\infty)$ with a single infinite element. The two point compactification makes distinctions between $\pm\infty$ so that one obtains the linear interval $[-\infty,\infty]$ without the endpoints being identified.

[2]The domain of $\psi(x,t)$ is taken as the Riemann sphere $\mathbb{S}^R=\mathbb{C}$ via the $(\hat{1},\hat{i})\to(\hat{x},ic\hat{t})$ correspondence.

As an example of an application for this new boundary condition, consider the axial current anomaly [132].[1] On qualitative grounds alone, one might suppose that the anomaly in the axial current is associated with a fundamental asymmetry in the underlying domain $\mathbb{S}^R = \mathbb{S}^2 \setminus \{\infty\}$ which ought to be symmetric. To the extent that quantum theory is said to live on the Riemann sphere, the subtraction of a 0-sphere rather than an asymmetric, lone point from $\mathbb{S}^2$ may have far reaching symmetry implications.

28 Boundary Terms at Infinity

Few phrases are repeated more often in physics than, "Integrating by parts and assuming that boundary terms at infinity go to zero..." As in the previous section, one usually restricts states to L^2:

$$\big|\psi\big\rangle \in L^2 \quad \Longrightarrow \quad \psi(\infty) = 0 \quad . \tag{28.1}$$

For any

$$\int_{-\infty}^{\infty} u\,dv = uv\bigg|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} v\,du \quad , \tag{28.2}$$

with u or v in L^2, we may conclude that $uv|_{-\infty}^{\infty}$ vanishes. Sometimes it is not clear that u or v are in L^2, and we still assume that the boundary vanishes. Furthermore, the integral's infinite bounds have not been studied in the framework of fractional distance. Thus, ignored boundary terms at infinity are an ideal place to discover new physics and methods for $\hat{M}^3$, $\widehat{\infty}$, and the neighborhood of infinity.

Consider the free field Lagrangian

$$\mathcal{L}(\phi) = \frac{1}{2}\Big[\big(\partial\phi\big)^2 - m^2\phi^2\Big] \quad . \tag{28.3}$$

The generator of the free field theory with source J is

$$Z = \int D\phi\, e^{i\int d^4x\{\frac{1}{2}[(\partial\phi)^2 - m^2\phi^2] + J\phi\}} \quad . \tag{28.4}$$

The first term $\mathcal{I}$ in the exponent's integral is solved with integration by parts. For

$$\mathcal{I} = \int d^4x\,\big(\partial\phi\big)^2 = uv\bigg|_{-\infty}^{\infty} - \int v\,du \quad , \tag{28.5}$$

[1] This anomaly may be demonstrated by the construction of an axial current operator from a pair of fields with origins separated by an ε which is later put to zero (the Adler–Bell–Jackiw formula [132].) One might explore cases in which the two fields' origins are located at $\widehat{0}$ and $\widehat{\infty}$, which are later identified.

we take

$$u = \partial\phi \qquad\qquad v = \phi$$
$$du = \partial^2\phi\, d^4x \ , \qquad \text{and} \qquad dv = \partial\phi\, d^4x \ , \tag{28.6}$$

so that

$$\mathcal{I} = \phi\,\partial\phi\Big|_{-\infty}^{\infty} - \int d^4x\,\phi\,\partial^2\phi \ . \tag{28.7}$$

Here, what may be the most-repeated phrase in quantum field theory sets the $\phi\,\partial\phi|_{-\infty}^{\infty}$ boundary term at infinity to zero due to the L^2 condition. We integrate the non-boundary term and plug the result back into Z to obtain

$$Z = \int D\phi e^{i\int d^4x\{\frac{1}{2}[-\phi\partial^2\phi - m^2\phi^2]+J\phi\}} = \int D\phi e^{i\int d^4x\{-\frac{1}{2}[\phi(\partial^2+m^2)\phi]+J\phi\}} \ . \tag{28.8}$$

Since so much of QFT depends on this integral and its permutations, we should very closely examine why we have set the boundary term in $\mathcal{I}$ as an identical zero. Specifically, we should examine whether or not this a ready place to add interactions between unequal levels of aleph, possibly by extending the bounds of integration beyond infinity or restricting the radius of the L^2 condition to the neighborhood of the origin:

$$\big|\psi\big\rangle \in \tilde{L}^2 \qquad \Longrightarrow \qquad \lim_{x\to\mathcal{F}_0}\psi(x) = 0 \ . \tag{28.9}$$

Physics requires that the probability amplitude for observing something at infinity is zero, but this neither precludes transfinite bounds of integration nor prevents a restricted $\tilde{L}^2$ radius. On the latter, the stated physical condition of a realistic potential for being observed is *better* said to require that ψ goes to zero at the end of the neighborhood of the origin. The MCM arithmetic axioms are such that we need only assume that physical fields go to zero at the outskirts of some local neighborhood of fractional distance, but there is no prohibition against them picking up again beyond that, especially when the observer's frame of reference will be transported beyond it in each application of $\hat{M}^3$.

The definition of Z in (28.4) is such that d^4x is over all of spacetime, not only a local neighborhood of fractional distance. Therefore, we must make explicit notation such that

$$\int_{\mathbb{R}} dx = \int_{-\infty}^{\infty} dx \quad \longrightarrow \quad \int_{\mathbb{R}_{\mathcal{X}}} dx = \int_{\mathcal{F}_{\mathcal{W}}}^{\mathcal{F}_{\mathcal{X}}} dx \ , \tag{28.10}$$

where $\mathcal{F}_{\mathcal{W}}$ and $\mathcal{F}_{\mathcal{X}}$ are sequential non-arithmatic numbers. An alternative notation developed for such cases in [2] is such that

$$\int_{\mathbb{R}_{\mathcal{X}}} dx \equiv \int_{\mathbb{R}(n)} dx = \int_{\mathcal{F}(n-1)}^{\mathcal{F}(n)} dx \quad . \tag{28.11}$$

Unfortunately, arithmetic is not defined among non-arithmatic numbers, so

$$\int_{\mathcal{F}(n-1)}^{\mathcal{F}(n)} dx = x \Big|_{\mathcal{F}(n-1)}^{\mathcal{F}(n)} = \mathcal{F}(n) - \mathcal{F}(n-1) = \text{undefined} \quad . \tag{28.12}$$

The likely resolution, as in [2], is that we must treat the non-arithmatic numbers on the n^{th} level of aleph as the natural numbers on the next higher level of aleph. The details of such a physical mechanism would be incorporated into the translation of the observer's frame onto a higher level of aleph (Section 1.6.5), or into a reimagined scheme for renormalization/regularization (Section 24). A mechanism for recasting $\mathcal{F}(n)\in\mathbb{F}$ as the $n\in\mathbb{N}$ on a higher level of aleph is easily conceptualized in the picture of the universe as a one quantum particle. The integral over all of spacetime written as an integral over multidimensional $\mathbb{R}_0\equiv\mathbb{R}(0)$ bounded by $\mathcal{F}(0)$ on a lower level of aleph will show up as an integral over one unit of volume on the higher level.

In [2], we have shown paradoxes related to the (n) enumeration scheme for the continuous spectrum of $\mathcal{X}$ in $\mathcal{F}_{\mathcal{X}}\in\mathbb{F}$. It is asked what must become of rigor if we are to label sequential elements of a continuum with integers? However, compared to the non-rigor in the infinite-dimensional path integral measure $D\phi$ and the much-loved but non-rigorous

$$\sum \delta t \quad \longrightarrow \quad \int dt \quad , \tag{28.13}$$

method, the slight abuse of (n) notation does not seem great. In each case, the hand-waving regards unallowed intermingling of countable and uncountable infinities. Physicists' affinity for taking such liberties is well contextualized in the non-rigor of writing the $\mathbb{R}_{\mathcal{X}}$ neighborhood as the n^{th} successive neighborhood $\mathbb{R}(n)$. However, further development of the paradoxes detailed in [2] (Section 7 therein) may lead to an enhanced understanding of (28.13), and in the foundations of calculus.

Part III: Problems in Mathematics

29 The Prime Number Theorem

The Riemann hypothesis (RH) is an important question in mathematics because it phrases a deeper question about the distribution of prime numbers. The negation of

RH was demonstrated by new methods for the neighborhood of infinity [2, 46, 47, 78], but the work was not carried through to the prime number application. Therefore, one would extend it to its consequences for prime numbers. Particularly, a well known formula involving the logarithmic integrals of ρ and $1-\rho$ looks amenable to plying with fractional distance analysis.

Limited work on the prime counting function $\pi(x)$ which has not been published shows that there are an infinite number of primes less than any number in the neighborhood of infinity. There exists an infinite number of primes in the neighborhood of the origin, so the divergence of the prime counting function evaluated at $x\in\widehat{\mathbb{R}}$ is the correct behavior. This work should be built upon and published. For prime numbers p, numbers of the form $\widehat{\infty}-p$ have the same distribution as the primes. Anything that can be learned about the distribution of $\widehat{\infty}-p$ will necessarily hold for the primes as well. While the distributions are the same, numbers in the neighborhood of infinity have slightly different arithmetic operations [2]. Given these new arithmetic tools, one might find new insights which were inaccessible across more than 150 years of analysis in the neighborhood of the origin. Particularly, the holy grail of number theory is a general algorithm for computing sequences of prime numbers, and this problem deserves attention. Therefore, a review of the prime number theorem and a survey its corollaries are in order.

30 The Riemann ζ Function in Quantum Theory

We have solved RH in [2] and elsewhere [46, 47, 78]. However, we have not gone on to treat the problem which made Riemann's hypothesis interesting: the prime number problem, as in the previous section. In the modern context for Riemann ζ function (RZF) problems, we have treated neither applications in cryptography nor Hamiltonian operators in quantum theory proportional to ζ. All of this work remains to be done, and the latter is the main topic of this section.

The connection of RH to the prime numbers is well known and concisely stated in many places, but the connection to quantum theory seems to be more like an intuition shared by a large number of well respected mathematicians and physicists. A survey of the evidence for an RZF-QFT connection is in order, and particularly a survey of the Hilbert–Pólya operator-based program for tackling RH. Burnol writes the following regarding that program [133].

> "[*We are convicted*] that the Riemann Hypothesis has a lot to do with (suitably envisioned) Quantum Fields. The belief in a possible link between

> the Riemann Hypothesis and Quantum Mechanics seems to be widespread and is a modern formulation of the Hilbert–Pólya operator approach. I believe that techniques and philosophy more organic to Quantum Fields will be most relevant. [T]his point of view has not so far led to success[.]"

If it is thought that studies in quantum theory might shed light on RH, then it is reasonable to expect that a solution to RH would shed light in the other direction. Regarding what can be extracted from the negation of RH in fractional distance analysis for applications in the arena of quantum theory, Borwein, Bradley, and Crandall write the following [134].

> "It is intriguing that any of the various new expansions and associated observations relevant to the critical zeros arise from the field of quantum theory, feeding back, as it were, into the study of the Riemann zeta function. But the feedback of which we speak can move in the other direction, as techniques attendant on the Riemann zeta function apply to quantum studies."

While attending an undergraduate non-linear dynamics course given by Cvitanović, the professor explained that he had become stuck in his research for a long time before discovering that his problem was equivalent to RH. He advised in all seriousness that if any students should ever run into a problem where they find themselves trying to prove the Riemann hypothesis, a change of research direction should be considered. Now that RH is negated, one would search for the application which depended on it.[1] While the exact mechanism by which ζ is connected to quantum chaos is not known to this writer, the following words from Berry and Keating [137], and then Brown [138], suggest that it is worth looking into. If so much association is seen by experts in the field, then it seems likely that the negation of RH would generate fruitful follow-on studies beyond the context in number theory.

> "Our purpose is to report on the development of an analogy, in which three areas of mathematics and physics, usually regarded as separate, are intimately connected. The analogy is tentative and tantalizing, but nevertheless fruitful. The three areas are eigenvalue asymptotics in wave (and particularly quantum) physics, dynamical chaos, and prime number theory. At the heart of the analogy is a speculation concerning the zeros of the Riemann zeta function (an infinite sequence of number encoding the

[1] Perhaps the application regards a theorem of Connes in non-commutative geometry that is equivalent to RH [135]. Most likely, the application can be found in Cvitanović's book [136].

primes): the Riemann zeros are related to the eigenvalues (vibration frequencies or quantum energies) of some wave system, underlying which is a dynamical system whose rays or trajectories are chaotic. Identification of this dynamical system would lead directly to a proof of the celebrated Riemann hypothesis. ***We do not know what the system is, but we do know many of its properties*** [*emphasis added*]."

"If you choose a number n and ask how many prime numbers there are less than n it turns out that the answer closely approximates the formula: $n/\log n$. The formula is not exact, though: sometimes it is a little high and sometimes it is a little low. Riemann looked at these deviations and saw that they contained periodicities. Berry likens these to musical harmonics: 'The question is what are the harmonics in the music of the primes? Amazingly, these harmonics or magic numbers behave exactly like the energy levels in quantum systems that classically would be chaotic.' This correspondence emerges from statistical correlations between the spacing of the Riemann numbers and the spacing of the energy levels. Berry and his collaborator Jon Keating used them to show how techniques in number theory can be applied to problems in quantum chaos and vice versa. In itself such a connection is very tantalizing. Although sometimes described as the Queen of mathematics, ***number theory is often thought of as pretty useless, so this deep connection with physics is quite astonishing***. [*emphasis added*] Berry is also convinced that there must be a particular chaotic system which when quantised would have energy levels that exactly duplicate the Riemann numbers. 'Finding this system could be the discovery of the century,' he says. It would become a model system for describing chaotic systems in the same way that the simple harmonic oscillator is used as a model for all kinds of complicated oscillators. It could play a fundamental role in describing all kinds of chaos. The search for this model system could be the holy grail of chaos... [*We*] cannot be sure of its properties, but Berry believes the system is likely to be rather simple, and expects it to lead to totally new physics. It is a tantalizing thought."

31 The Hodge Theater and Anabelomorphy

Joshi writes the following [139].

"I coined the term anabelomorphy as a concise way of expressing 'Mochizuki's anabelian way of [*doing things*].'"

The purpose of the problem in this section is to identify a 2012 leap in Mochizuki's program in inter-universal Teichmüller theory (IUT) [140–143] as a rebranding of the MCM. The unit cell was not published until 2013, but the nine year process of revision leading to the journal publication of [140–143] in 2021 may have incorporated later MCM work. To begin, one notes that the first figure in the first IUT paper [140] (Figure 24) is quite like the 2009 time-wrapped-around-a-cylinder idea for MCM time periodicity [31]. (This was later supplanted by periodicity in unit cell [7].)

We will phrase the main criticism of Scholze and Styx (SS) against Mochizuki's claimed proof of the ABC conjecture [144] as pertaining to a fundamental concept in unit cell. Paraphrasing, SS [144] have refused to acknowledge that Mochizuki has proven the ABC conjecture because he takes isomorphic objects as unequal. To the extent that Mochizuki's "Hodge Theater" is only the MCM unit cell dressed in inaccessible jargon, we will motivate the existence of two isomorphic objects which are not the same object in the sense of abstract algebra. Although the domain of the wavefunctions $\psi(x^i)$ in each $\mathcal{H}'_n$ is just a Euclidean 3-space E^3, each E^3 is the spacelike slice of Minkowski space at a given x^0. Therefore, although all infinite Euclidean 3-spaces are isomorphic copies of E^3, we may distinguish among them by labeling them with the affine parameter x^0. They are unequal. In other words,

$$k \neq j \qquad \Longrightarrow \qquad \mathcal{H}_k \neq \mathcal{H}_j \quad . \tag{31.1}$$

It seems likely that this can be parlayed into a rebuttal of SS' criticism if Mochizuki has utilized the MCM without introducing errors.

Joshi describes similtude to the MCM in [139].

> "One could think of anabelomorphy in the following picturesque way: One has two parallel universes (in the sense of physics) of geometry/arithmetic over p-adic fields K and L respectively. If K, L are anabelomorphic (i.e. $K \neq L$) then there is a worm-hole or a conduit through which one can funnel arithmetic/geometric information in the K-universe to the L-universe through the choice of an isomorphism of Galois groups $GK \simeq GL$, which serves as a wormhole. Information is transferred by means of amphoric quantities, properties and alg. structures. The K and L universes themselves follow different laws (of algebra) as addition has different meaning in the two anabelomorphic fields K, L (in general.) As one might expect, some information appears unscathed on the other side, while some is altered

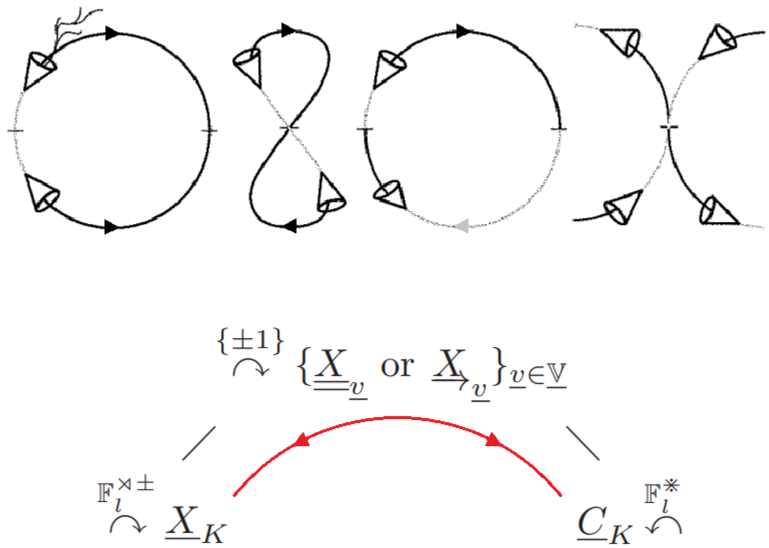

Figure 24: The upper figure adapted from [31, 39] derives correspondence between cosmological bouncing and particle scattering by wrapping the time axis of Minkowski space around a cylinder and then imposing smooth deformations. Horizontal hashes mark big bounces. The lower algebraic diagram from [140] (red curve added) contextualizes Mochizuki's work in the 2009-2012 period when MCM time periodicity was obtained by imposing cylindrical topology. It is suggested that Mochizuki has condensed the possibilities for various inter-bounce modules (above) into a single algebraic diagram (below).

by its passage through the wormhole. Readers will find ample evidence of this information funneling throughout this paper (and also in [*Mochizuki's papers*] which lay the foundations to it.)

"I hope that these results will convince the readers that Mochizuki's idea of anabelomorphy is a useful new tool in number theory with many potential applications (one of which is Mochizuki's work on the abc-conjecture.) Especially it should be clear to the readers, after reading this paper, that assimilation of this idea (and the idea of anabelomorphic connectivity) into the theory of Galois representations should have interesting consequences for number theory. Here I have considered anabelomorphy for number fields but interpolating between the number field case and my observation that perfectoid algebraic geometry is a form of anabelomorphy, it seems reasonable to imagine that anabelomorphy of higher dimensional fields will have applications to higher dimensional algebraic geometry as well."

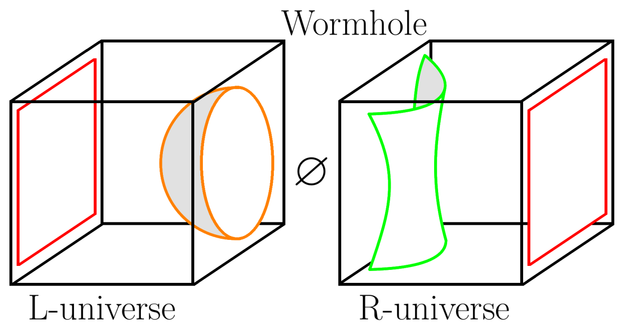

Figure 25: This figure casts the description of anabelomorphy given by Joshi in [139] (Section 1.7 therein) in terms of the unit cell.

Here, the reader's attention is called to what Joshi hopes will "be clear to readers." It is suggested that the "ample evidence" cited by Joshi is evidence of the foundations of Mochizuki's later work in the MCM. The language of L- and K-universes due to Joshi [139] signifies the left and right $\Sigma^{\pm}$ universes under a clever change of notation $R \to K$, as in Figure 25. Although the citations were removed in the above excerpt, Joshi cites 2013 and 2015 works of Mochizuki in addition to the four principle papers from 2012 [140–143]. It must be determined if Joshi's L, K notation references something Mochizuki had done before the 2013 publication of the non-cylindrical unit cell in 2013 [7].

Clarifications made by Mochizuki following a series of IUT-related discussions in 2018 seem to reflect MCM developments in the intervening years which were not contained in the 2012 papers themselves. Mochizuki writes the following in [145]. (The formatting is altered, and most citations are removed.)

> "Another topic to which a substantial amount of time and energy was devoted, especially during the first few days of the March discussions, was the topic of labels to distinguish distinct copies of various familiar objects that play substantively different roles in the various apparatuses treated in IUTch. SS (especially, Scholze) were substantially opposed to the use of labels in IUTch. This opposition appeared to be based, to a substantial extent, on 'taste/aesthetics.' In this context, however, it should be remembered that in fact 'labels' [*are*], in effect, situations in which one wishes to distinguish distinct copies of various familiar objects that play substantively different roles within a complicated apparatus. [...]
>
> "In light of the general considerations concerning the use of labels [...], it is of interest to review the way in which labels for distinct copies of various

familiar objects are employed in IUTch in order to construct apparatuses that play various substantive roles in IUTch that cannot be achieved if the labels are deleted. One fundamental example of this phenomenon is the bookkeeping apparatus for labels for evaluation points within a single Hodge theater.

"This phenomenon is discussed in detail in *[140]*, §I1 (and indeed throughout *[140]*!). On the other hand, such labels within a single Hodge theater were only mentioned very briefly during the March discussions. The 'label issues' that were discussed in substantial detail during the March discussions concern the labels [...] that correspond to the [*Hodge theaters*] in the log-theta-lattice. Here, we begin our discussion of these labels by recalling the (highly noncommutative!) diagram that is used to denote the entire log-theta-lattice [...], together with the portion of the log-theta-lattice [...] which consists of the vertical arrows in the 0- and 1-columns, together with the single horizontal arrow between the [*Hodge theaters.*]

"[*A Hodge Theater is*] a single model of the conventional ring/scheme theory surrounding the elliptic curve over a number field under consideration. One then considers two types of gluing (denoted by the vertical and horizontal arrows in the diagrams) between certain portions of the Hodge theaters in the domain and codomain of each arrow. The vertical arrows denote log-links, while the horizontal arrows denote θ-links."

If the Hodge theater is the unit cell, the evaluation points are certainly the labeled branes. The multiple "vertical arrows" seem to refer to the $\{x^0_+, x^0, x^0_-\}$ chronological times in the labeled branes while the "single horizontal arrow" must refer to χ^4. It should be investigated to what extent such language may have appeared in Mochizuki's 2012 papers pre-dating the unit cell, and to what extent these arrows might have referred to the MCM's arrow-laden unit cell precursors [31,39]. It is likely that Mochizuki's context for elliptic curves is derived from the 2011 $\hat{M}^3$ operator which we have used to arrive at elliptic curves in Section 1.11.5. Therefore, we have laid the foundation for a large work unit sifting through a thousand or more pages of Mochizuki's notoriously inaccessible jargon and unfortunate typographical choices. One would attempt to *see what he did there* and scan for any new insights that may have been included in the voluminous obfuscating layers. Such insights may or may not exist, which is to say that Mochizuki may have taken this writer's idea and not added anything before rebranding it at his own idea.

32 Regularity Structures

> "Martin Hairer takes $3m Breakthrough prize for work a colleague said must have been done by aliens." [54]

After Hairer won the Fields Medal in 2014, Quastel said his work in regularity structures must have been done by aliens because he knows very well that a "regularity structure" is the MCM unit cell equipped with the $\hat{M}^3$ operator. Consider Hairer's words in [53].

> "The purpose of this article is to develop a general theory allowing to formulate, solve and analyse solutions to semilinear stochastic partial differential equations of the type
>
> $$\mathcal{L}u = F(u, \xi) \quad , \tag{32.1}$$
>
> where $\mathcal{L}$ is a (typically parabolic but possibly elliptic) differential operator, ξ is a (typically very irregular) random input, and F is some nonlinearity."

Hairer describes the general problem of $\hat{M}^3$ which was the topic of Section 1. He uses $\mathcal{L}$ as $\hat{M}^3$ and condenses everything we don't know about the physics of the unit cell into F. Hairer continues as follows [53].

> "One major difference between the results presented in this article and most of the literature on quantum field theory is that the approach explored here is truly non-perturbative and therefore allows one to deal also with some non-polynomial equations [...]. We furthermore consider parabolic problems, where we need to deal with the problem of initial conditions and local (rather than global) solutions. Nevertheless, the mathematical analysis of QFT was one of the main inspirations in the development of the techniques and notations presented [*elsewhere in [53]*].
>
> "Conceptually, the approach developed in this article for formulating and solving problems of the type [*(32.1)*] consists of ***three steps*** [*emphasis added*].
>
> 1. In an algebraic step, one first builds a 'regularity structure', which is sufficiently rich to be able to describe the fixed point problem associated to [*(32.1)*] Essentially, a regularity structure is a vector space that allows to describe the coefficients in a kind of 'Taylor expansion' of the solution around any point in space-time. The twist is that the

'model' for the Taylor expansion does not only consist of polynomials, but can in general contain other functions and/or distributions built from multilinear expressions involving ξ.

2. In an analytical step, one solves the fixed point problem formulated in the algebraic step. This allows to build an 'abstract' solution map to [*(32.1)*]. In a way, this is a closure procedure: the abstract solution map essentially describes all 'reasonable' limits that can be obtained when solving [*(32.1)*] for sequences of regular driving noises that converge to something very rough.

3. In a final probabilistic step, one builds a 'model' corresponding to the Gaussian process ξ we are really interested in. In this step, one typically has to choose a renormalisation procedure allowing to make sense of finitely many products of distributions that have no classical meaning. Although there is some freedom involved, there usually is a canonical model, which is 'almost unique' in the sense that it is naturally parametrized by elements in some finite-dimensional Lie group, which has an interpretation as a 'renormalisation group' for [*(32.1)*].

"We will see that there is a very general theory that allows to build a 'black box', which performs the first two steps for a very large class of stochastic PDEs. For the last step, we do not have a completely general theory at the moment, but we have a general methodology, as well as a general toolbox, which seem to be very useful in practice."

Step one regards the construction of the unit cell. In step two, Hairer defines the metric and 4-potential in $\mathcal{H}$ from "reasonable" limits in $\Sigma^{\pm}$. Step three is the main problem which remains open in the MCM: how to push Schrödinger evolution from the $\mathcal{H}$-brane, through $\varnothing$, and then into the forward $\mathcal{H}$-brane in a way that agrees with experiment? Consider this writer's words from [30].

"There are varying philosophies on quantum experimentation so let us define a process thoroughly. Two measurements must be made, A and B. The boundary condition set by A will be used to predict the state at B. The observer applies physical theory to trace a trajectory [*from* A] into the future and [*to*] predict what the state will be at that time. Before the observer can verify the theory, sufficient time must pass that the future event occurs. Once this happens a signal from the event reaches the observer in the present and a second measurement B becomes possible. From the present we predict into the future. In time that becomes the past. When the signal from that event reaches the observer a theory can be tested. A three-fold process.

$$\text{Present} \;\mapsto\; \text{Future} \;\mapsto\; \text{Past} \;\mapsto\; \text{Present}\;\text{.''} \tag{32.2}$$

The Fields Medal and $3M award suggest that Hairer has sufficiently developed the mathematical foundations of the issues raised in Section 1 to the point where serious things can be said about their resolutions.[1] Therefore, one would conduct of survey of Hairer's main results regarding "regularity structures." New insights achieved by Hairer, if any are found, may be useful for pushing the MCM past certain conceptual hurdles.

The reader is encouraged to carefully note that Hairer's 2013 publication date for [53] comes chronologically later in the literature than the first iterations of $\hat{M}^3$ *and* the unit cell [3, 7, 30, 39]. Indeed, Hairer's March 2013 publication date following so closely after the unit cell was published in January 2013 [7] is *oddly* timed with Ellis' and You's fallacious exaggeration regarding reasonable doubt in March 2013 [28]. One might entertain the notion that Ellis, You, Hairer, and others were working as agents of a conspiracy to besmirch and naysay the MCM while plagiarizing it. If such a conspiracy exists, as is suggested in Appendix C, it is unlikely that this writer has uncovered all of the evidence in the literature. However, it remains that Hairer's work has at least the appearance of being well received in the mathematical community, and one would survey the work looking for new insights that might have applications in the MCM. Such insights may or may not exist. Hairer may have taken this writer's idea and not added anything at all before rebranding it at his own idea.

33 Quantum Set Algebra

Finkelstein's final seven uploads to arXiv all appear to be MCM response papers [146–152]. The last two are mathematical in nature and support this problem's placement in Part III, in part. Before making a brief summary of the points of interest in Finkelstein's arXiv publications, including his work on quantum set algebra, we will make a contextual aside regarding the MCM's *administrative* peer review status.

Finkelstein had already moved into professor emeritus status when this writer began PhD studies at Georgia Tech. This was unfortunate because Finkelstein's

[1] Another reasonable conclusion to draw from Hairer's $3M award is that the main purpose of the Breakthrough Prize created by Milner, an Israeli, is to elevate the position of those who have an interest in the scientific demise of this writer. Perhaps similar things can be said about the Fields Medal committee.

research area overlaps with this writer's interests. After [31] was censored by the arXiv moderators in 2009, and after [39] was similarly forbidden in 2011, this writer reached out to the faculty in the School of Physics and was put in contact with Finkelstein. After a few brief conversations, Finkelstein assured this writer that a fair hearing of peer review could be had IJTPD. At that time, it was not known to this writer that Finkelstein was an editor at IJTP from 1977 to 2005. In hindsight, the "fair hearing" was only that Finkelstein would don his anonymous reviewer's mask to unleash all the criticisms he had withheld in conversations meant only to milk this writer's ideas without offering his own constructive inputs. After submitting a manuscript regarding the MCM and the theory of reverse time,[1] the reviewer quickly denied publication. The brief rejection letter is accurately paraphrased as, "The author doesn't know the ADM theorem[2] from a hole in the ground." It is assumed that the reviewer was Finkelstein, and it is certain that a fair hearing was not had.

A rebutting response to the editor cited an assumption of cosmological isotropy and homogeneity in the Arnowitt–Deser–Misner (ADM) model of the universe as a non-orientable manifold. This assumption is called the cosmological principle. From that assumption, ADM extract the differential element of surface area at spacelike infinity and use that as the centerpiece of their theorem [40]. However, modern data which was only published near the end of Finkelstein's long career shows that the cosmological principle is not sound. Correlations in the structure of the CMB are not consistent with the isotropy used by ADM. The quadruple and octupole moments seen in CMB fluctuations are aligned when they have no reason to be. Both moments are further aligned with the plane of the solar system along what is called an *axis of evil* in modern cosmologies where the universe very much has an inherent orientation [153]. On the other hand, the two-universe structure cited in the manuscript submitted to IJTPD [39] was consistent with a symplectic 2-form at spacelike infinity from which either positive- or negative-definiteness of the universe's p^0 would follow (Section 44). This directly and cleanly refuted the reviewer's (Finkelstein's) criticism about the ADM theorem preventing a negative energy universe moving backward in time from a cosmogenesis event.

If a fair hearing would have been had at IJTPD, the point in the rebuttal would have been acknowledged, but it was ignored, and the manuscript was removed from the online submission system about two weeks later. The true events were that the ideas in the MCM blew away anything Finkelstein had done in his career with respect

[1] The manuscript submitted to IJTPD was a version of [39].

[2] This theorem proving the positive-definiteness of the p^0 component of the universe's 4-momentum was said to disallow another universe with $p^0 < 0$. A rebuttal to this claim appears in Section 44.

to original thinking.[1] When his attempt to pontificate and detract from behind an anonymous reviewer's mask was destroyed on its merits due to new data which he had not appreciated, or whose consequences for the the ADM theorem he had not evaluated, he retreated into a fog of anonymity to remove the manuscript from consideration. He did not acknowledge that new data is inconsistent with the cosmological principle which was in vogue across the several decades of his professional life. The entire encounter with Finkelstein, in person and anonymously, reeks of egotism and academic treachery. Although citations in the remainder of this section will demonstrate that Finkelstein had already written at least two MCM response papers before meeting with this writer in 2011 [146,147], he pretended to gross ignorance in our conversations. Furthermore, he withheld his criticism about the ADM theorem during our meetings. This writer had never heard of the ADM theorem before the critique at IJTPD, but the wrongful reliance on the cosmological principle was identified in less than hour once the reviewer's citation was received, as was the workaround described in Section 44. Had Finkelstein mentioned the ADM theorem in person, it would have been impossible for him to ignore the rebuttal. However, he shrewdly withheld his criticism until it would be possible to ignore rebuttals and avoid any forced admission of wrongness from the safety of a zero-accountability environment.

Hitler writes the following [155].

> "The more I debated with them the more familiar I became with their argumentative tactics. At the outset they counted upon the stupidity of their opponents, but when they got so entangled that they could not find a way out they played the trick of acting as innocent simpletons. Should they fail, in spite of their tricks of logic, they acted as if they could not understand the counter arguments and bolted away to another field of discussion. They would lay down truisms and platitudes; and, if you accepted these, then they were applied to other problems and matters of an essentially different nature from the original theme. If you faced them with this point they would escape again, and you could not bring them to make any precise statement. Whenever one tried to get a firm grip on any of these apostles one's hand grasped only jelly and slime which slipped through the fingers and combined again into a solid mass a moment afterwards. If your adversary felt forced to give in to your argument, on account of the observers present, and if you

[1] Eddington–Finkelstein coordinates are due to Penrose [154] and ought to be called Penrose coordinates because neither Eddington nor Finkelstein ever wrote them down. They came to be named as they are because Penrose, when he was receiving accolades for his coordinates, cited much older papers written by Eddington and Finkelstein as providing the *ideation* for his own paper [154]. Therefore, it is the height of irony that Finkelstein's claim to fame is based on an acknowledgment of progenitive ideation of the sort Finkelstein himself was so keen to avoid.

> then thought that at last you had gained ground, a surprise was in store for you on the following day. The Jew would be utterly oblivious to what had happened the day before, and he would start once again by repeating his former absurdities, as if nothing had happened. Should you become indignant and remind him of yesterday's defeat, he pretended astonishment and could not remember anything, except that on the previous day he had proved that his statements were correct. Sometimes I was dumbfounded. I do not know what amazed me the more—the abundance of their verbiage or the artful way in which they dressed up their falsehoods. I gradually came to hate them."

Finkelstein never admitted to this writer that he was the reviewer at IJTPD, but perhaps he gained mastery in such tactics of evasiveness and deceit during his time at the Hebrew University and Yeshiva University. The reviewer simply laid down a platitude regarding the ADM theorem before slipping away without being forced to concede the cosmological principle's unsound footing in modern experiments or the implied possibility for a symplectic 2-form at spacelike infinity. Indeed, Finkelstein employed such cunning that he was not forced even to concede awareness of the existence of the rebuttal. The reviewer at IJTPD wrote his criticism sounding as if he knew what he was talking about, but he did not. Instead, he counted on the stupidity of his critique's readers to be such that the contents of the ADM paper [40] would not be verified. When this writer's rebuttal was submitted in short order, the reviewer bolted away and used his authority to remove the manuscript from consideration. Furthermore, the reviewer's implication that the ADM theorem should take precedence over *the law of conservation of momentum* was profoundly stupid.

The following appears in Finkelstein's [146]. Published in 2010, we suggest [146] is a response to an MCM paper published in 2009 [31].

> "The proposed quantum theory, termed recursive, represents the system as a recursive quantum assembly. Its modules have Fermi-Dirac statistics, and are modularizations, or unitizations, of like assemblies of a lower level, or rank. Each assembly is also interpreted as a quantum topological simplex with its constituent modules as its vertices."

It is suggested that Finkelstein refers to Figure 26 which first appeared in [31]. The quantum theory is recursive due to its placement in the context of Ashtekar's model of loop quantum cosmology. The modules are said to have Fermi–Dirac statistics because the particles in the Feynman diagram of Figure 26 obey such statistics. So,

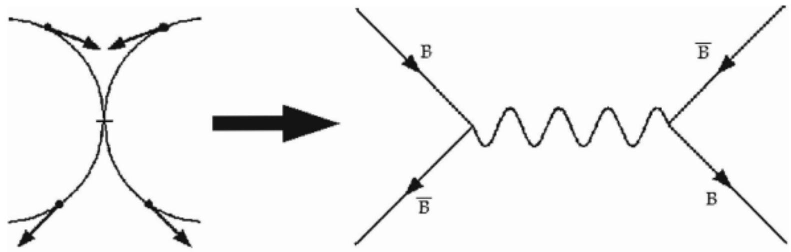

Figure 26: In this figure taken from [31], time increases toward the right (negative time increases toward the left.) On the left, a spacetime diagram has been deformed so as to impose a periodic boundary condition along the x^0 axis. The same diagram of another universe whose time arrow points oppositely intersects the former at a big bounce. It was suggested that the familiar Feynman diagram for electron-positron scattering might offer a ready framework for describing the cosmological mechanics of such a bounce complex. A further implication derived by replacing the spacetime picture with a particle picture is that we may associate the anomalous, non-zero, positive baryon number B of the universe with the +1 lepton number of an electron, and likewise for the anti-universe and positron.

although Finkelstein found non-compliance with the ADM theorem to be an unfixable problem when he was hiding behind an anonymizing bureaucracy in 2011, the man himself saw enough merit in the MCM to take it as his own without citation in 2010. At the end of the abstract to [146], Finkelstein writes, "The gauge structure, the spin-statistics correlation, the space-time metric, and the Higgs field are modeled." This an apparent reference to the final sentence of the abstract in [31]: "No attempt at quantification is made." Since Finkelstein claims to have moved forward with quantification, his results must be surveyed. If they are found to be useful, they must be incorporated into future work.

In [147] (August 2011), Finkelstein writes the following.

> "Present-day quantum field theory can be regularized by a decomposition into quantum simplices. This replaces the infinite-dimensional Hilbert space by a high-dimensional spinor space and singular canonical Lie groups by regular spin groups. It radically changes the uncertainty principle for small distances. Gaugeons, including the gravitational, are represented as bound fermion-pairs, and space-time curvature as a singular organized limit of quantum non-commutativity."

The uncertainty principle is said be changed at small distances because Finkelstein is making an appeal to Ashtekar's "repulsive force of quantum geometry." By this so-called force, Ashtekar has claimed that topological singularities are avoided during big bounce events. Instead of absolutely divergent collapse, the repulsive force of quantum geometry somehow kicks in at very small length scales so as to avoid the formation of a pointlike singularity. It is now this writer's opinion that the repulsive force of quantum geometry is nothing more than an artifact of Ashtekar's numerical algorithms and Finkelstein was bluffing to suggest that he understood a new method in "present-day QFT." However, Finkelstein further claims to represent gaugeons (bosons) as bound fermion pairs which may be useful for associating the baryon number of a universe in a spacetime diagram with the lepton number of a fermion in a Feynman diagram. Certainly, Finkelstein's remark pertains to Figure 26. Perhaps his faith in the ADM theorem led him to discount the existence of a reversed universe so that fermion pairs on both sides of an annihilation event were associated with one universe on either side of a bounce. Perhaps the fermion pairs were an ingoing particle and an outgoing one corresponding to a universe going into the bounce and coming out of it, and Finkelstein fully copied the dual universe structure in 2010 before disputing it on the basis of the ADM theorem in 2011.

After meeting with this writer following a second event of censorship at arXiv in 2011, and before a third similar event at IJTPD, Finkelstein left Atlanta for a period of weeks claiming to have traveled to a certain castle in Bulgaria. Upon returning, he published [148–150] on arXiv in quick succession. These must be considered response papers to [30,39] (November 2011). One wonders if Finkelstein used a so-called retreat in Bulgaria as cover for travel to Israel where he would more closely collaborate with others among this writer's most detractive antagonists.

The first statement of the three-fold process for $\hat{M}^3$ appeared in [30]: observation, prediction, waiting, and observation again. Finkelstein writes the following in [148] which followed [30, 39] by about two months.

> "Call the process, if any, by which natural laws are formed 'logogenesis'. Josephson proposed that quantum observer-participation leads to logogenesis. [...] In the logogenesis proposed by Peirce, nature first acts by chance, then acts form habits, and finally habits harden into more permanent laws. The formation and hardening of habits are not further described by Peirce. I speculate next on a still unformulated quantum logogenesis with elements of those of Einstein and Peirce. Peirce's 'habit-forming tendency of nature' can be read as a remarkable premonition of Bose statistics. In each step in time, the system is first annihilated and then recreated. This was asserted by Islamic Scholastics of 10th century Baghdad and is explicit

> in quantum field theory, where a creation ψ^* follows every annihilation ψ in the action principle for a particle.

Finkelstein has dubbed the process for $\hat{M}^3$ a process of "logogenesis." He refers to a system of logogenesis based on observations, exactly like $\hat{M}^3$. Later in [148], he writes, "Peirce's signs occur in a semantic triangle of sign, interpretant, and object." This seems to be an attempt to attribute ideation for his own *forthcoming* three-fold process of logogenesis to work other than what is found in [30]. Keeping in mind that the unit cell was not yet devised in 2012, the three steps of $\hat{M}^3$ were $t_0 \to t_{\max} \to t_{\min} \to t_0$ which is the reference made by Finkelstein's inclusion of creation and annihilation in each step of logogenesis. The universe falls into a big crunch at $t_{\max}$, and then it is reborn in a big bang at $t_{\min}$.

Finkelstein writes the following in [149].

> "A finite relativistic quantum space-time is constructed. Its unit cell has Palev statistics defined by a spin representation of an orthogonal group. When the Standard Model and general relativity are physically regularized by such space-time quantization, their gauges are fixed by nature; the cell groups remain."

Fixation of the gauge by nature is a clear reference to the FSC result in [30]. The finite quantum spacetime follows the MCM program of modular spacetimes [39] and references the 2D box used in the original derivation of α_{MCM} [30]. When meeting with this writer upon his return from "Bulgaria," Finkelstein was exceedingly insistent that Palev statistics [149] were what the MCM was lacking. Therefore, Palev statistics should be taken with a grain of salt. Deliberate misinformation should be considered. Finkelstein's character suggests that he would offer bad advice as a complement to the constructive criticism he withheld while feigning *total unfamiliarity* with the MCM in late 2011. However, Finkelstein's insistence on writing his own variant of the MCM in the language of statistics throughout [146–152] can probably be trusted as the best intuition of a man who spent his life searching for a better theory. Palev statistics should be examined on its merits, and a statistical treatment of the MCM should be developed regardless of the utility or non-utility of such statistics.

Finkelstein's citation of "di-fermions" as "Palevons" [149] suggests that he was at least as enamored of the mechanism in Figure 26 as was this writer. Di-fermions are the representation of the pre- and post-bounce states given bosonic baryon numbers as pairs of fermions. The high esteem of Finkelstein should further undermine detractors' insistence that the correspondence, or *duality*, between the spacetime and particle pictures (Figure 26) was vague, meaningless, and/or other unscholarly things.

Figure 27: This figure excerpted from [150] shows the three steps of $\hat{M}^3$.

Finkelstein writes the following in [150].

> "I once asked Jack Schwartz what the difference was between mathematics and physics. At the time both were just equation-juggling to me. He was strap-hanging homeward from Stuyvesant High School, where we had just met, and he answered by drawing a hat in the subway air with his free hand: [*Figure 27.*] He explained that the bottom line is the real world and the top line is a mathematical theory. At its left-hand edge we take data from the real world and put them into a mathematical computation, and at the right-hand side we compare the output of the computation with nature. The loop closes if the theory is right. This diagram also applies to quantum systems, if the statistical nature of quantum theory is taken into account. Then the bottom line is not one experiment on the system but a statistical population of them. The question remains of what the symbols of mathematics mean to a mathematician. Some decades later I asked Jack Schwartz what '1' means, and he replied that it means itself. This took me aback. I had not considered that possibility. Symbols generally mean something not themselves. Memorandum:
>
> $$1 = \text{`1'} \quad ." \tag{33.1}$$

By the conspicuous absence of the present writer's name in the gratitude section at the end of [149], perhaps Finkelstein would have his readers believe that without seeing [30, 31, 39], he spontaneously decided to write a paper [150] about how his old friend Jacky Boy from back in the day had explained to him 70 years ago that any mathematical description of a physical process is constrained to follow $\hat{M}^3$. Perhaps the Jackster never told him any such thing, and this lie was part of a ruse designed to avoid recognizing the keen insights had by this writer.

After the unit cell [7] and MCM particle scheme [6] were published in 2013, Finkelstein produced two final uploads to arXiv regarding "quantum set algebra" [151, 152]. The following are excerpted from [151].

> "Quantum field theory can be physically regularized by modularizing it on several levels of aggregation. [...] Relativistic locality makes each point of a spacelike surface a separate physical system, in that the variables it carries are independent of those of any other point of the surface. Therefore a relativistic theory is necessarily a many-system theory. In a quantum theory this implies an algebra of creation and annihilation operators, not a mere vector space. The Hilbert space theory is a one system theory. Its constructs correspond to those of classical predicate algebra, with no analysis into independent systems."

> "In the Standard Model or any other quantum field theory [...], an experiment is a network of operations of quantum creation and annihilation, or input and output. For brevity, call these *port operations*, portations, or most briefly ports, and say that they *import* or *export* quanta. [...] Boole's Laws of Thought and the set theory of Cantor, in which 'A set is a Many that allows itself to be thought of as a One,' explicitly concern mental processes. Adapted to quantum physical processes, the Laws of Thought of Boole become Laws of Ports and the set of Cantor becomes a Many that can be ported as One."

The modularized, many-system theory refers to the 2013 brane structure of the unit cell [7]. Finkelstein seems to have associated the chirological interval separating branes with a spacetime diagram's spacelike interval. Along side the first statement of the unit cell, a brief note restating the rebuttal to Finkelstein's criticism regarding the ADM theorem appeared in [7]. Finkelstein may have associated a possible symplectic 2-form at spacelike infinity with a doorway or "port" out of $\mathcal{H}$, into the chirological interval. References to "the Laws of Thought" and "mental processes" allude to the MCM process for $\hat{M}^3$ being psychological in nature.

Finkelstein writes the following in [152].

> "A modular quantum architecture is given for the space-time, particles, and fields of the Standard Model and General Relativity. It assumes a right-handed neutrino[.]"

Finkelstein alludes to the 2013 MCM particle scheme [6] following from the architecture of the unit cell. The reference to handedness invokes the distinction between left- and right-handed spacetime quanta distinguishing quark and lepton pairs. Finkelstein also writes the following [152].

> "Because the Heisenberg indeterminacy principle is so weakened, it can no longer be excluded that gaugeons are pairs of odd quanta, though these odd quanta are not necessarily the ones able to exist as free quanta."

The MCM particle scheme is such that the gauge bosons are formed from pairs of "odd quanta," or fermions. Finkelstein does not cite the MCM particle scheme. Instead, he cites some vague weakening of the Heisenberg indeterminacy principle. Indeed, Finkelstein had already suggested such a construction in 2011 [147], based on Figure 26 apparently, without the benefit of the explicit model of bosons as pairs of fermions given in [6].[1]

Overall, the dating of the main body of citations in [146–152] suggest that Finkelstein stopped learning new things at some point in the 1970s. The prose in the papers is abrupt if not broken, and the jargon is idiosyncratic or anachronistic to the point of being non-standard (which is ok.) However, since these papers represent an honest attempt to steal the MCM, the work should be dissected, and any original contributions due to Finkelstein should be utilized in future work, if any are found to exist.

34 The Navier–Stokes Equation

The Navier–Stokes problem posed by the Clay Mathematics Institute asks if there exist "physically reasonable" solutions to the Navier–Stokes equation [156]. Often one solves complicated differential equations with combinations of exponential functions, so the utility of fractional distance analysis towards new zeros for e^x must be evaluated in the context of this problem. In the neighborhood of the origin, e^x has no roots on the real line. In the neighborhood of infinity, e^x has an infinite number of roots. This behavior should allow a rich new class of solutions for differential equations. The big exponential function E^x [2] offers another tool which may be useful for finding new solutions to differential equations. As the Riemann hypothesis was immediately solved with fractional distance analysis, almost trivially [2, 46, 47], the similar mathematical structure of the Navier–Stokes problem demanding nothing more than a solution to an equation suggests that this problem might be solved in another forthright manner.

[1] This writer had not yet begun to review Finkelstein's publications when the MCM model of particles was constructed in [6].

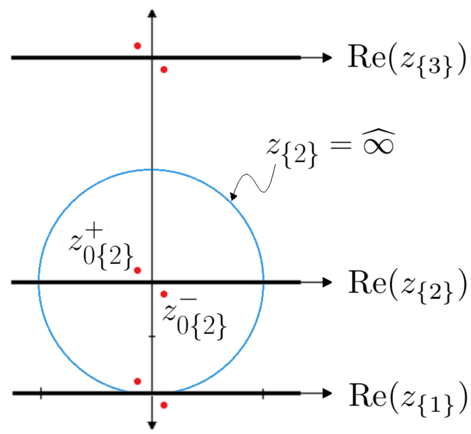

Figure 28: This figure adapted from [71] shows a proposal for a proof of the existence of the Yang–Mills mass gap. The red dots are the poles of the k_0 part of the free propagator on a given level of aleph. Horizontal lines show the real axis of $\mathbb{C}$ on the $\{k\}$ level of aleph. Such instances of $\mathbb{C}$ are joined by mutual transverse continuations onto $\mathbb{C}^*_\pm$. The blue circle is the radius at infinity relative to the origin of $\mathbb{C}_{\{2\}}$. The radius at infinity increases as $\{k\}$ increases. The mass gap is attributed to the enclosure of three poles where only two are usually considered.

35 The Yang–Mills Mass Gap

The Millennium Prize regarding Yang–Mills theory [157, 158] requires proof that a quantum Yang–Mills theory exists in four dimensions, and that it contains the mass gap $\Delta > 0$ required by QCD. An MCM solution to this problem was proposed in [71]. Briefly, a transfinite continuation of the complex plane was introduced and associated with the changing level of aleph attendant to $\hat{M}^3$, as in Figure 28. (We will call this continuation $\mathbb{C}^*$ though its context in [71] was not exactly as described in Section 1.2.4.) In Figure 28, $z_{\{k\}}$ is the complex variable on the k^{th} level of aleph. The origin of each $z_{\{k\}} \in \mathbb{C}_{\{k\}}$ is found where the real axis crosses the vertical axis. Using QED as a jumping off point for this more challenging problem in QCD, one often examines the free propagator $D(x-y)$ from which we integrate out the time part $D^0(x-y)$. The two poles of $D^0(x-y)$ are the pairs of red dots shown in Figure 28: $z^\pm_{0\{k\}}$ on the k^{th} level of aleph such that

$$z^\pm_{0\{k\}} \equiv z^\pm_0 \in \mathbb{C}_{\{k\}} \quad . \tag{35.1}$$

Although we have taken a transfinite extension of $\mathbb{C}$, we preserve the notion of an integration path at infinity. (The blue circle is the path at infinity relative to the

$k = 2$ level of aleph. The radius of the path at infinity increases as the level of aleph increases.) To simplify $D^0(x-y)$, one usually employs the Cauchy residue theorem, integrates along the real axis, and closes the Cauchy C curve with a path at infinity in the upper or lower complex half plane. Depending on the path, one pole or the other is enclosed, the integral at infinity vanishes, and the integral along the real axis is left to equal to $2\pi i$ times the enclosed residue.

The idea for generating a mass gap is found with the third pole near the origin of $\mathbb{C}_{\{k-1\}}$ which is included within the path at infinity in $\mathbb{C}^*$, but it does not exist in $\mathbb{C}$. Appealing to the more complicated structure of QCD requiring such features as quark confinement which have no analogues in QED, one might take the function in the Cauchy theorem to be a sum of propagators on the k and $k-1$ levels of aleph. We associate the $\mathbb{C}^*_\pm$ extension of $\mathbb{C}$ out of $\mathcal{H}$ in the $\chi^4_\pm$ directions with QCD but not QED because χ^4 is associated with quarks but not leptons (Section 0.3). This thinking may provide guidance for the problem regarding the existence of a quantum Yang–Mills theory, but presently we aim to describe the mass gap. In $\mathbb{C}^*_\pm$, a third pole is enclosed by the integration path, and the sum of the enclosed residues will not be zero. The two poles near the origin of $\mathbb{C}_{\{k\}}$ will cancel, but there will be a small remainder due to the pole on the lower level of aleph. One would attempt to correlate this structure with the existence of the QCD mass gap. Furthermore, one will obtain different values when integrating around the upper or lower complex half-planes.

36 The Banach–Tarski Paradox and Information Density

Consider the series

$$x = 1 + 2 + 4 + 8 + \ldots \quad , \tag{36.1}$$

such that

$$x = 1 + 2\big(1 + 2 + 4 + \ldots\big) = 1 + 2x \qquad \Longrightarrow \qquad x = -1 \quad . \tag{36.2}$$

To avoid this result, one often makes a heuristic argument regarding the manipulations of infinite series. However, fractional distance analysis offers new analytical tools with which to define a measure of information density such that the contradiction in (36.2) would be avoided because the parenthetical expression has one less term in it than the series in (36.1). Namely, x has $\widehat{\infty}$ terms in it, but the parenthetical expression in (36.2) only has $\widehat{\infty} - 1$ terms.[1] Therefore, the parenthetical expression cannot be

[1]Earlier MCM work in [70] relied on such distinctions between series with $\widehat{\infty}$ implicit terms and with $\widehat{\infty} \pm 1$ implicit terms. The extra term at the end was described as a qubit, and applications of information density toward quantum theory were discussed. Mainly, the inner products of infinite series with odd or even numbers of terms were shown to have "qubit" remainders.

exactly equal to x, and the false implication does not follow. Such a statement of information density is not possible when $1 + 2 + 4 + ...$ has ∞ terms in it. New tools which make it possible to quantify this notion of information density should be advanced to the state of some formal treatment.

In 1924, Banach and Tarski (BT) published a set-theoretical decomposition of the unit sphere [159]. They showed that pointwise operations on the set of a sphere's points may be executed such that the recombined points constitute two equal spheres. While some claim that there is no paradox because BT were correct to show that one sphere's points can be used to construct a second, equal sphere, the paradox is that one does not equal two. For instance, one might define the natural numbers in units of spheres. A set with one sphere in it is the number one, etc. Dividing one sphere into its fractional parts should not yield fractions that recombine to more than one sphere. Still, this is the result obtained by BT. No errors have been found in their derivation, so it is called a paradox. To avoid the paradox, one would invoke information density. As an illustration, consider BT's step where the set of terminal up rotations is made congruent to the sets of terminal up, left, and right rotations. After canceling the up operation with a down operation, the information density would be reduced, and the strict equality would be avoided. The $1=2$ paradox is avoided when the naturals are defined in units of spheres *with a certain information density.* As the final two spheres have lower information density than the initial sphere, they cannot quantify the same units as the initial sphere. It remains to formally recompose the result of Banach and Tarski in the language of fractional distance analysis.

37 The Topology of the Real Line

In [2], we have gone to great lengths to define a topology for $\mathbb{R}$. In the opinion of this writer, it is likely that the work can be extended to show that the topology of $\mathbb{R}$ is $\mathbb{S}^0$: the zero sphere. The argument proceeds as follows.

First, [2] meticulously defines the downward representation of geometric objects in algebraic language, but little attention is given to the upward representation of algebraic objects with geometric language. We know it is possible to put an infinite number of algebraic points into a geometric point, but the reverse relationship is not yet determined. By the general similitude of geometric points and algebraic points (numbers), one expects that algebraic points may contain an infinite number of geometric points. If this reciprocity is proven or axiomized, it should be possible to prove that $\mathbb{R}$ has the $\mathbb{S}^0$ topology. In other words, $\mathbb{R}$ may be represented as two points.[1]

[1] A 2-sphere is a hollow ball in 3D space, a 1-sphere is a circle in the plane, and a 0-sphere is two points in a line.

Treating only the non-negative branch of $\mathbb{R}$ for simplicity, one would use the result of [2] that geometric points may contain algebraic intervals to resolve geometric points A and B as the algebraic neighborhood of the origin and the maximal neighborhood of infinity respectively:

$$A \equiv [0, \mathcal{F}_0) \quad , \quad \text{and} \qquad B \equiv [\mathcal{F}_{\widehat{\infty}-1}, \infty) \quad . \tag{37.1}$$

Recall that $\mathcal{F}_\mathcal{X} \in \mathbb{F}$ is the supremum of the $\mathbb{R}_\mathcal{X}$ neighborhood of fractional distance. One would convert the intervals to geometric line segments as

$$[0, \mathcal{F}_0) \equiv AA' \quad , \quad \text{and} \qquad [\mathcal{F}_{\mathcal{X}_\text{max}}, \infty) \equiv B'B \quad , \tag{37.2}$$

and then proceed to iteratively construct the intermediate algebraic neighborhoods of infinity from A' and B'. It must be examined whether the limit of infinite iterations can be used to show that $\mathbb{R}$ has the 0-sphere topology.

38 The Twin Primes Conjecture

The program described here is highly speculative relative to the work described in other sections. That being stated, one might endeavor to prove the twin primes conjecture as follows. (This approach was first suggested in [160].) Require that the orthogonality of plane waves on different levels of aleph ultimately follows the small box normalization convention (Section 1.7.3) in which the chirological wavenumber β is quantized:

$$\psi_n(x, t, \chi^4) = \exp\big\{ i\big(kx - \omega t + \beta_n \chi^4\big)\big\} \quad , \tag{38.1}$$

Place $\mathcal{A}$ and Ω on the $k \pm 1$ levels of aleph relative to $\mathcal{H}_k$. Assume the chirological wavenumber in $\mathcal{H}_k \neq \mathcal{H}_0$ is the difference of contributions from $\mathcal{A}$ and Ω:

$$\beta_{k+1} - \beta_{k-1} = \beta_k \quad . \tag{38.2}$$

Develop a framework in which β_k increases by Φ on successive levels of aleph so that

$$\beta_{k \pm n} = \Phi^{\pm n} \beta_k \quad . \tag{38.3}$$

Substituting (38.3) into (38.2) yields

$$\Phi \beta_k - \varphi \beta_k = \beta_k \qquad \Longrightarrow \qquad \Phi = 1 + \varphi \quad . \tag{38.4}$$

This must be true when $\beta_k = \Phi^k \beta_0$, so we obtain a general relationship

$$\Phi^{k+1} = \Phi^k + \Phi^{k-1} \quad . \tag{38.5}$$

This is known to be satisfied by the golden ratio Φ. One would find an example in which $k \pm 1$ are twin primes, as would be the case for $\mathcal{H}_4$. It is a general idea in the MCM that certain non-classical effects in $\mathcal{H}_k$ should be attributed to contributions from other levels of aleph, e.g.: anti-gravity in mechanical precession (Section 15), and we will attempt to prove the twin primes conjecture by the invariance of $\mathcal{H}_k$ under $k \to k'$. Such a proof might proceed by induction, but we will give a more specific procedure.

Following a logical program like the sieve of Eratosthenes, one would develop a requirement that contributions from all non-prime levels of aleph are totally attenuated in the bulk lattice; only contributions from prime levels of aleph may contribute to physics in $\mathcal{H}_k$. One appeals to the fundamental theorem of arithmetic for an appropriate mechanism. Likewise, the Euler product form of the RZF sketches a path by which one is able to eliminate non-prime numbers. Under the given conditions, one would invoke the translational invariance of $\mathcal{H}_k$ for any k to conclude that there must exist an infinite number of twin primes. In other words, if the nodes of a rectangular progression in the golden ratio are associated with a pair of twin primes, then the infinite continuation of the golden spiral will imply an infinite number of such primes because $\Phi^{k+1} = \Phi^k + \Phi^{k-1}$ for any k. As this pertains to the chirological wavenumber, one would find that there must exist infinite twin primes because physics in $\mathcal{H}_k$ cannot depend on the absolute value of k.

39 The Limits of Sine and Cosine at Infinity

The results

$$\lim_{x\to\infty} \sin(x) = 0 \quad , \qquad \text{and} \qquad \lim_{x\to\infty} \cos(x) = 1 \quad , \tag{39.1}$$

derived in [161] rely on $\widehat{\infty}$ having multiplicative absorption but not additive absorption. This has several undesirable implications for basic arithmetic and does not reflect the conventions of the by now mature framework for fractional distance analysis in $\mathbb{R}$ [2]. Therefore, the result should be revisited under the arithmetic axioms which remove both of the additive and multiplicative absorptive properties from $\widehat{\infty}$ [2]. It is expected that the result will hold up when the immeasurable, non-arithmatic numbers $x \in \mathbb{F}$ serve as some regularized boundary condition along $\mathbb{R}$ such that the behavior of sine and cosine on approach to $\widehat{\infty}$ along $\mathbb{R}^+$ is the mirror image of the behavior on egress from the origin. However, further analysis is required to determine whether the result will hold up under the general arithmetic axioms in [2].

If the result will survive, applications toward integrals with infinite bounds and similar problems should be developed. For instance, the Dirac δ function has an integral definition

$$\delta(x) = \frac{1}{2\pi}\int_{-\widehat{\infty}}^{\widehat{\infty}} dk\, e^{ikx} = \frac{1}{2\pi i x} e^{ikx}\bigg|_{-\widehat{\infty}}^{\widehat{\infty}} = \frac{1}{\pi x}\frac{e^{ix\widehat{\infty}} - e^{-ix\widehat{\infty}}}{2i} = \frac{1}{\pi x}\sin(\aleph_x) \quad . \tag{39.2}$$

By the assumed translational invariance of trigonometry functions among neighborhoods of fractional distance, $\sin(\aleph_x)=0$ because $\sin(0)=0$. If sine is equal to zero at the center of one neighborhood of fractional distance, it should be equal to zero in the center of all of them. Note the agreement of the assumed $\sin(\aleph_x)=0$ with

$$\sin(\aleph_x) = \frac{e^{ix\widehat{\infty}} - e^{-ix\widehat{\infty}}}{2i} = \frac{\left(e^{i\widehat{\infty}}\right)^x - \left(e^{-i\widehat{\infty}}\right)^x}{2i} = \frac{1-1}{2i} = 0 \quad , \tag{39.3}$$

where $e^{\pm i\widehat{\infty}}=1$ follows from (39.1). If we do not invoke

$$\lim_{x\to 0^{\pm}} \frac{1}{x} \quad \longrightarrow \quad \pm\widehat{\infty} \quad , \tag{39.4}$$

then the final step of (39.2) is undefined for $x=0$. This blow up leads to the $\delta(0)=\infty$ property. To use (39.4), we should examine the product of $(\pi x)^{-1}$ with the series decomposition of sine:

$$\begin{aligned}
\lim_{x\to 0}\frac{1}{\pi x}\sin(\aleph_x) &= \lim_{x\to 0}\frac{1}{\pi x}\sum_{n=0}^{\infty} c_n \aleph_x^{2n+1} \\
&= \lim_{x\to 0}\frac{1}{\pi}\sum_{n=0}^{\infty} c_n \aleph_x^{2n}\frac{\aleph_x}{x} \\
&= \lim_{x\to 0}\frac{1}{\pi}\sum_{n=0}^{\infty} c_n \aleph_x^{2n}\aleph_1 \quad .
\end{aligned} \tag{39.5}$$

The $x\to 0$ limit of $\aleph_x$ requires deeper analysis in the ε–δ framework, as in Section 1.6.9. We have defined $\aleph_0=0$ in [2], but $\aleph_\varepsilon$ is greater than any natural number for any real $\varepsilon>0$. This suggests we might take

$$\lim_{x\to 0}\aleph_x = \mathcal{F}_0 \quad , \tag{39.6}$$

where $\mathcal{F}_0$ is the least positive non-arithmatic number. Since $\mathcal{F}_0>1$, (39.5) will agree with $\delta(0)=\infty$:

$$\lim_{x\to 0}\frac{1}{\pi}\sum_{n=0}^{\infty}c_n\aleph_x^{2n}\aleph_1=\frac{1}{\pi}\sum_{n=0}^{\infty}c_n\aleph_{\mathcal{F}_0^{2n}}\approx\infty \quad . \tag{39.7}$$

On the other hand, if the limit is zero, the resulting expression contains $0\times\aleph_1=\aleph_0=0$, and we do not immediately obtain the correct behavior for $\delta(x)$ at $x=0$. Such nuance remains to be analyzed. A vast ocean of similar problems will be opened up to new analysis by proof of the limits of sine and cosine at infinity under the axioms of fractional distance analysis.

40 The Cauchy Residue Theorem

The coefficient of the Cauchy residue theorem

$$\oint_C dz\, f(z)=2\pi i\sum \mathrm{Res} f(z) \quad , \tag{40.1}$$

contains three of the four ontological numbers. One would attempt to make an extension of complex analysis to $\mathbb{C}_{\pm}^{*}$ in the form

$$\oint_C dz\, f(z)=2\pi i\Phi^{\Delta k}\sum \mathrm{Res} f(z) \quad , \tag{40.2}$$

where k refers to a level of aleph and C acquires a winding number so that it can begin on one level of aleph and end on another. Δk is the change in the level of aleph between the start and endpoints of C, so $\Delta k=0$ gives the usual formula. While it remains to be determined why the exponent on Φ should be different than those on 2, π, and i, one understands that $\hat{\Phi}$ is unique for pointing in the direction which allows us to separate the start and end points of C on successive $\mathcal{H}$-branes. In that case, we slightly abuse the $\oint$ closed path integral notation.

The neighborhood of infinity provides rich new structure around poles. One might envision the poles of a function piercing an infinite number of complex planes such that each intermediate neighborhood of infinity crossed on the approach to the pole is resolved on $\mathbb{C}_{\{k\}}$: the complex plane on the k^{th} level of aleph.

41 Intermediate Numerical Scale

Although fractional distance analysis is strictly a subset of real analysis, this problem calls for a survey of alternatives to $\mathbb{R}$ such as the surreals [162] and hyperreals [163]. In either system, division by an infinite quantity can yield an infinitesimal but never a finite number. Fractional distance was conceived in part to fill this intermediate scale gap with numbers in the neighborhood of infinity such that

$$\frac{\aleph_{\mathcal{X}}}{\infty} = \mathcal{X} \quad . \tag{41.1}$$

For $\mathcal{X} \in (0,1)$, numbers such as $\aleph_{\mathcal{X}}$ occupy an intermediate numerical scale between infinites and the naturals such that

$$n \in \mathbb{N} \qquad \Longrightarrow \qquad \frac{n}{\infty} = 0 \quad . \tag{41.2}$$

Intermediate scale was relied upon heavily in the architecture for a negation of the Riemann hypothesis in [48]. Briefly, an infinitesimal neighborhood around a point and a smaller, nested *hypercomplexly infinitesimal* neighborhood were associated with adjacent odd and even levels of aleph with respect to finite numbers on a third adjacent even level. By scale invariance under shifting levels of aleph, the requirement for two tiers of infinitesimals was found to imply two tiers of finites: numbers in the neighborhood of the origin and those in the neighborhood of infinity. Namely, the scale progression

$$\text{finite} \quad \longrightarrow \quad \text{infinitesimal} \quad \longrightarrow \quad \text{hypercomplexly infinitesimal} \quad , \tag{41.3}$$

was found to imply a similar progression

$$\text{infinite} \quad \longrightarrow \quad \text{intermediate} \quad \longrightarrow \quad \text{finite} \quad , \tag{41.4}$$

under a shift of the level of aleph by two. Aside from [48], another discussion of this requirement may be found in [164]. Given the absence of an intermediate scale in popular extensions of $\mathbb{R}$ containing infinitesimals, one would survey those models attempting to bridge the gap to fractional distance analysis.

Part IV: More Problems in Physics

Many problems in Part IV are concisely presented as surveys of others' work with an eye toward MCM connections. Miscellaneous topics in cosmology appear here as well.

42 Randall–Sundrum Models

This writer became aware of Randall–Sundrum (RS) models in the years after developing the unit cell. The likeness of these models to the MCM is striking. The main purpose of the work described here will be to cast the MCM unit cell, to the extent that it may be possible, as a third type of RS model beyond the primary RS1 and RS2 models [165, 166].[1]

The following is excerpted from [167].

> "Randall–Sundrum models (also called 5-dimensional warped geometry theory) are models that describe the world in terms of a warped-geometry higher-dimensional universe, or more concretely as a 5-dimensional anti-de Sitter space where the elementary particles (except the graviton) are localized on a (3 + 1)-dimensional brane or branes. The two models were proposed in two articles in 1999 by Lisa Randall and Raman Sundrum because they were dissatisfied with the universal extra-dimensional models then in vogue. Such models require two fine tunings; one for the value of the bulk cosmological constant and the other for the brane tensions. Later, while studying RS models in the context of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, they showed how it can be dual to technicolor models. The first of the two models, called RS1, has a finite size for the extra dimension with two branes, one at each end. [165] The second, RS2, is similar to the first, but one brane has been placed infinitely far away, so that there is only one brane left in the model. [166].
>
> "The model is a braneworld theory developed while trying to solve the hierarchy problem of the Standard Model. It involves a finite five-dimensional bulk that is extremely warped and contains two branes: the Planckbrane (where gravity is a relatively strong force; also called 'Gravitybrane') and the Tevbrane (our home with the Standard Model particles; also called 'Weakbrane'). In this model, the two branes are separated in the not-necessarily large fifth dimension by approximately 16 units (the units based on the brane and bulk energies). The Planckbrane has positive brane energy, and the Tevbrane has negative brane energy. These energies are the cause of the extremely warped spacetime. In this warped spacetime that is only warped along the fifth dimension, the graviton's probability function is

[1] The first citation of Sundrum and Randall in [165] is to a paper of Arkani-Hamed and Dvali [5] regarding new dimensions near one millimeter. This paper was cited here in Sections 0.1 and 15 as agreeing with and supporting the characteristic scale for new MCM physics at 10^{-4}m. RS state that their own work is similar to the model presented in [5].

extremely high at the Planckbrane, but it drops exponentially as it moves closer towards the Tevbrane. In this, gravity would be much weaker on the Tevbrane than on the Planckbrane."

RS1 is AdS_5 bounded by two branes separated by a finite distance across which spacetime is *warped* in the fifth dimension only. RS warp factor is identical to MCM scale factor, so this is very similar to the MCM scheme for increasing scale along χ^4. However, the warp factor is not like the continuum of increasingly curved, maximally symmetric MCM branes because the RS warp factor acts uniformly on the entire 4D part of the metric. In RS models, the warping is said to be caused by the energies of the branes, but the MCM approach defines scale as a property of a quantum operator algebra whose analytical underpinnings have not yet been determined. The MCM construction leaves energetic considerations to follow as a consequence when the usual program in physics is that everything should follow from the energy landscape. The alternative, usual view in RS models may be useful for answering questions left open when energy is said to result in the MCM rather than to cause.

Randall and Sundrum (RS) write the following in [165].

> "We propose that the metric is not factorizable but rather the four-dimensional metric is multiplied by a 'warp factor' which is a rapidly changing function of an additional dimension. The dramatic consequences for the hierarchy problem that we identify [...] follow from the particular non-factorizable metric,
>
> $$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2 \quad , \tag{42.1}$$
>
> where k is a scale of order the Planck scale, x^μ are the coordinates for the familiar four dimensions, while $0 \leq \phi \leq \pi$ is the coordinate for an extra dimension, which is a finite interval whose size is set by r_c."

RS put the warp factor directly into the metric, but MCM scale factor is associated with renormalization of the observer's reference frame onto the level of aleph of a given brane. After non-unitary evolution by $\hat{M}^3$, unitarity is restored by the scale factor in the brane at the end. The example in Section 0.2 defined unit scale in the $\mathcal{H}$-brane by an equation in the form

$$g_{\mu\nu} = \Phi g^+_{\mu\nu} - \varphi g^-_{\mu\nu} \quad , \tag{42.2}$$

where Φ and $-\varphi$ play the role of the RS warp factor $e^{-2kr_c\phi}$ on the branes where we take $g^\pm_{\mu\nu}$. RS achieve unit scale where $\phi = 0$ (the Planckbrane), and later in their paper they add an absolute value so the warp factor becomes $e^{-2kr_c|\phi|}$. This is said to cause "the graviton wavefunction" to fall off exponentially quickly away from the Planckbrane, which RS call the *hidden* brane. The hierarchy problem is said to result because the graviton wavefunction has fallen off so greatly by the time it reaches the *visible* Tevbrane at $\phi=\pm\pi$. The absolute value on $|\phi|$ makes RS physics symmetric about the Planckbrane, but scale is not symmetric around MCM branes. This is an important distinguishing feature among RS and MCM models. The main difference between them is that the metric warp factor is the kernel of RS physics, but we have used the KK metric to impose unification of EM and gravitation while putting a warp factor-analogue into the non-unitarity of $\hat{M}^3$. Kaluza–Klein theory is the kernel of the metric part of the MCM, and a complete metrical analysis remains to be carried out. Namely, the relative merits of taking $g^{\pm}_{\mu\nu}$ in (42.2) as the 4D parts of $g^{\pm}_{AB}$ in the low $\chi^4_{\pm}$ limits or in the Ω- and $\mathcal{A}$-branes must be fully evaluated. The detailed metrical analyses in [165, 166] provide a template of important cases and considerations.

The method by which r_c sets the scale of RS' $\phi\in[0,\pi]$ parameter (actually $\phi\in[-\pi,\pi]$) is the same one by which an arbitrary chronological time between measurements is normalized around $\chi^4\in[-\varphi,\Phi]$. Where r_c is a constant in RS theory, however, the MCM r_c equivalent will take a unique value in each unit cell because the time interval between measurements may be irregular. In the limit of vanishing $A^{\mu}_{\pm}$, the MCM metric

$$g^{\text{MCM}}_{AB}=\begin{pmatrix} g_{\alpha\beta}+\chi^4 A_\alpha A_\beta & \chi^4 A_\alpha \\ \chi^4 A_\beta & \chi^4 \end{pmatrix}\quad , \tag{42.3}$$

has line element

$$ds^2_{\text{MCM}}=g_{\alpha\beta}d\chi^\alpha d\chi^\beta+\chi^4\big(d\chi^4\big)^2\quad , \tag{42.4}$$

where $g_{\alpha\beta}=\eta_{\mu\nu}$ is implicit for comparison with (42.1). An overall scale factor is implicit as well. The MCM metric reduces to the KK metric through $\chi^4=\phi^2$ for a scalar field ϕ. In (42.1), constant r_c^2 replaces the KK scalar field.

RS continue as follows [165].

> "Because our spacetime does not fill out all of five dimensions, we need to specify boundary conditions, which we take to be periodicity in ϕ, the angular coordinate parameterizing the fifth dimension, supplemented with the identification of (x,ϕ) with $(x,-\phi)$ [...]. We take the range of ϕ to be from $-\pi$ to π; however the metric is completely specified by the values in the range $0\leq\phi\leq\pi$. The orbifold fixed points at $\phi=0,\pi$ will be taken

as the locations of two 3-branes, extending in the x^μ directions, so that they are the boundaries of the five-dimensional spacetime. The 3-branes can support (3+1)-dimensional field theories. Both couple to the purely four-dimensional components of the bulk metric:

$$g_{\mu\nu}^{\text{vis}}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = \pi) \quad , \quad g_{\mu\nu}^{\text{hid}}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = 0) \quad , \tag{42.5}$$

where G_{MN}, $M, N = \mu, \phi$ is the five-dimensional metric. This set-up is in fact similar to the scenario of [*Arkani-Hamed et al. when laying out the case for new dimensions near* $10^{-3}m$ *in [5].*]"

RS introduce a 5D metric G_{MN} whose $\mu, \nu \in \{0, 1, 2, 3\}$ submetrics $G_{\mu\nu}$ correspond to 4D metrics on the bounding branes of the 5D space: the hidden Planckbrane and the visible Tevbrane. Similarly, we have the $g_{AB}^{\pm}$ metrics for 5D abstract coordinates in $\Sigma^{\pm}$ and 4D physical metrics $g_{\alpha\beta}^{\pm}$ on the bounding branes. $g_{\mu\nu}^{\text{vis}}$ is like the metric on $\mathcal{H}$ (the Tevbrane) and $g_{\mu\nu}^{\text{hid}}$ is like the metric on a hidden $\mathcal{A}$-, Ω-, or $\varnothing$-brane. RS assign unit scale factor (warp factor) to the hidden brane whereas we associate unit scale with quantum mechanical unitarity and, thus, the visible $\mathcal{H}$-brane. However, RS will go on to explain that their model works with either of the Tev- or Planckbranes placed at $\phi = 0$.

RS' identification of their boundaries in 5-space as 3-branes shows that they are not considering the timelike part of their AdS_5 space as part of the bulk enclosed by the boundaries. This is another significant departure from the MCM even while it contextualizes what is meant when each $\mathcal{H}_k$-brane corresponds to a measurement at an observer's proper chronological time t_k. t_k identifies a 3-brane within $\mathcal{H}_k$, but we will only say that the observer's proper time was t_k when he was in $\mathcal{H}_k$ without reducing the dimensionality of the boundary. Furthermore, RS only consider $\{-++++\}$ 5-space while we consider the $\{-+++\pm\}$ topologies such that complexity is introduced when 3-branes in Euclidean signature $\{+++\}$ can be time evolved in more than one way. One envisions modularized dynamical MCM process in which the time evolutions of spacelike 3-branes are alternatingly chronological and chirological, and wherein we might achieve arbitrage of information around time loops. Processes like $t \to -it$ Wick rotation might be used to reorient MCM 3-branes with respect to x^0 or χ^4 such that lower case Latin metric counting indices are shifted from $\{0, 1, 2, 3\}$ to $\{1, 2, 3, 4\}$ resulting in non-factorizable mechanisms beyond those considered by RS.

RS continue as follows [165].

"Until this point, we have viewed $M \approx M_{\text{Pl}}$ as the fundamental scale, and

> the TeV scale as a derived scale as a consequence of the exponential factor appearing in the metric. However, one could equally well have regarded the TeV scale as fundamental, and the Planck scale of 10^{19} GeV as the derived scale. That is, the ratio is the physical dimensionless quantity. From this viewpoint, which is the one naturally taken by a four-dimensional observer residing on the visible brane, the large Planck scale (the weakness of gravity) arises because of the small overlap of the graviton wave function in the fifth dimension (which is the warp factor) with our brane. In fact, this is the only small number produced. All other scales are set by the TeV scale."

Here, RS point out that the warp factor is such that either of the Planckbrane or the Tevbrane may be taken as the fundamental brane. Likewise, we may represent the unit cell centered on $\mathcal{H}$ or $\varnothing$. The MCM scale progression in Φ^k suggests that any brane can be taken as the fundamental one with unit scale by a change of variables $k \to k'$. The RS branes' locations at $\phi = 0$ and $\phi = \pm\pi$—two opposite poles of a circle—imply that we should be able to continue the RS 5-space past the Tev- or Planckbrane, depending on which is placed at $\phi = 0$, by unwrapping ϕ from around a cylinder. In RS1 however, the behavior by which the absolute value in (42.6) causes the graviton wavefunction to fall of exponentially quickly away from the Planckbrane cannot be preserved if the Tevbrane is taken as the fundamental scale *and* the fifth coordinate is still continued beyond the Planckbrane. For instance, consider a rescaling $r_c\phi \to y$ so that (42.1) becomes

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \quad , \quad \text{with} \qquad y \in \mathbb{R}_0 \quad . \tag{42.6}$$

If the Tevbrane is placed at $y = 0$ and y is continued beyond $y = \pm r_c\pi$, then what RS call the graviton no longer falls off exponentially quickly away from the Planckbrane. To cast the MCM as an RS model, it is required to continue y as stated so as to impose an asymmetric scale factor around branes, but this adversely affects the freedom to take the Tevbrane as the fundamental brane. In turn, this should affect MCM freedom to center the unit cell on $\mathcal{H}$ or $\varnothing$. By allowing the fifth coordinate y to extend beyond the $\phi = \pm\pi$ bounding branes, RS1 can be stepped toward congruence with the MCM, but this breaks another desirable correspondence elsewhere in the model. The extent to which this latter correspondence may be important or required for casting the unit cell as an RS model must be examined. Since symmetry in the $\phi = \pm\pi$ boundaries appears hard-coded in RS models, one might consider two different ϕ-symmetric RS braneworlds and then seek to construct an MCM unit cell as their piecewise union.

The following is excerpted from [167].

> "The RS2 model uses the same geometry as RS1, but there is no TeV brane. The particles of the standard model are presumed to be on the Planck brane. This model was originally of interest because it represented an infinite 5-dimensional model, which, in many respects, behaved as a 4-dimensional model. This setup may also be of interest for studies of the AdS/CFT conjecture. [...] In 1998/99 Merab Gogberashvili published on arXiv a number of articles on a very similar theme *[168–170]*."

The Tevbrane ($\mathcal{H}$) is absent in the RS2 model. Sometimes it is said that it is moved to infinity whereas the Tevbrane is separated from the Planckbrane by a finite distance in RS1. Considering that the MCM introduces two semi-infinite 5-spaces to induce 4D physics on the boundary between them, one would join two RS2 models as $\Sigma^{\pm}$ such that both models' Tevbranes at infinity become a shared MCM $\mathcal{H}$-brane (which is not included in either of $\Sigma^{\pm}$.) In another union of RS models, one would simply concatenate $\mathcal{H}$ to one of $\Sigma^{\pm}$ so that one 5D space has two bounding branes (RS1), and the other has only one (RS2). The one where $\mathcal{H}$ lies at infinity would be said to reside on a higher or lower level of aleph. The RS1/RS2 union is also interesting because it matches the globally open and closed 4D topologies in the slices of $\Sigma^{\pm}$ despite RS1 and RS2 each using the $\{-++++\}$ 5D topology. Since the RS branes are only 3-branes, RS models of both types may be amenable to embedding in the pseudo-Lorentzian $\{-+++-\}$ topology of Σ^{-}.

The following is excerpted from [171].

> "The Randall–Sundrum model [165, 166] is a class of string theory inspired models in combined cosmology and particle physics, which assume that the observable universe constitutes the asymptotic boundary of an ambient anti de Sitter spacetime: the force of gravity would pertain to the full anti de sitter 'bulk' spacetime, but the gauge fields and fermion matter fields would be constrained to reside on that boundary, as would hence be all observations made via electromagnetic radiation by observers inside this cosmology. Hence the extra bulk dimensions in these models need not be small (technically: the fiber spaces need not be compact topological spaces with tiny Riemannian volume) in order to be unobservable for observers. This is in contrast to the (historically much older) Kaluza–Klein compactification models for physics with extra dimensions. Therefore Randall–Sundrum-like models are also referred to as large extra dimension models."

RS make a similar comment in [166] about their model differing from comparable models by not requiring compactification of dimensions greater than four. Since the MCM scale increases with increasing levels of aleph, it will be prudent to compare it to existing models of large extra dimensions.

43 Brans–Dicke Theory

Scalar-tensor theories are ones in which gravitation is controlled by a rank-2 metric tensor and also a varying scalar field. Brans–Dicke theory [172, 173] is the most prominent example of such a theory. The MCM's KK metric contains a scalar field ϕ, so the body of literature on such scalar-tensor connections should be surveyed. Brans' review of scalar fields in physics [174] is likely to contain various insights and tools useful for describing the MCM's 5D abstract and 4D physical metrics, and the connections between them. Particularly, the generalized Brans–Dicke theory is obtained by converting the Ricci scalar R to a generalized function [175], and this is similar to what we have done by identifying the Ricci scalar $R \propto (\ell_\pm^2)^{-1}$ with the KK scalar field via $R = \phi^2 = f(\chi_\pm^4)$.

44 The Arnowitt–Deser–Misner Theorem

A 1960 theorem of Arnowitt, Deser, and Misner (ADM) proves that the p^0 component of the universe's 4-momentum is positive-definite [40]. This result forbids a second universe with $p^0 < 0$ such that the total energy of two universes with opposite time arrows sums to zero. However, this is required for conservation of momentum in cosmogenesis, so the MCM requires modifications in the underpinnings of this theorem.

In [40], ADM suppose that the universe is rotationally and translationally invariant (g_{ik} is isotropic.) Then their theorem proceeds in the analysis of non-orientable manifolds. However, modern astrophysical experiments show anomalous multipole correlations in the temperature fluctuations of the cosmic microwave background (CMB). Furthermore, the CMB appears warmer to the north of the plane of the solar system and cooler to the south, and the already mutually correlated CMB quadrupole and octupole moments are further aligned with the axis of this heat distribution. Krauss said the following [176].

> "The new results are either telling us that all of science is wrong and we're the center of the universe, or maybe the data is simply incorrect, or maybe it's telling us there's something weird about the microwave background results and that maybe, maybe there's something wrong with our theories on the larger scales."

In an obvious way, if the distance from an observer to a surface is the same in any spatial direction, the CMB at about 14Gcy for example, then the surface is a sphere, and the observer is at its center. This fact places the Earth at the center of the universe. Data from experiments such as WMAP [177] and Planck [178] suggest this arrangement. Such data runs contrary to ADM's assumption that the universe should be translationally and rotationally invariant on large scales. Land and Magueijo have called this structure the "axis of evil" because it greatly confounds longstanding and well loved models of cosmology such as the one used in the ADM theorem. Therefore, we have reason to discount the isotropic g_{ij} assumption in the original ADM theorem. Many subsequent, alternative derivations of the ADM theorem have appeared in the intervening decades, and a survey of such work is required to show that any theorem which forbids $p^0<0$ necessarily fails in the general case.

A possible workaround for avoiding the theorem's implication proceeds as follows. ADM assign a differential element of area to the surface at spacelike infinity in the form [40]

$$dS_i = \frac{1}{2}\,\varepsilon_{ijk}dx^j dx^k \quad . \tag{44.1}$$

However, orientable manifolds may be equipped with symplectic forms on their boundaries. The consequence of ADM's work may be avoided if one rejects (44.1) in favor of

$$dS_i = \frac{1}{2}\,\varepsilon_{ijk}dx^j \wedge dx^k \quad . \tag{44.2}$$

The $dx^j \wedge dx^k = -dx^k \wedge dx^j$ property of the wedge product nullifies the ADM theorem when the opposite sign carries through to allow positive- or negative-definite energy. Although the Levi–Civita symbol reverses the sign with permutations of j and k, it cannot be determined if the positively signed case in (44.1) should correspond to $dx^j \wedge dx^k$ or $dx^k \wedge dx^j$.

Since the universe $\mathcal{H}$ connects to $\Sigma^{\pm}$ via some exotic geometry, one would examine the cases for symplectic 2-forms at spacelike infinity. If it can be demonstrated absolutely that (44.2) is the proper surface element, then the implication following from (44.1) will be avoided, as is required.

45 The Borde–Guth–Vilenkin Theorem

The Borde–Guth–Vilenkin (BGV) theorem is said to rule out any model of cosmology in which time extends infinitely far into the past. In [56], BGV give a simple argument proving that a past spacetime boundary must exist for expanding spacetimes with metric

$$ds^2 = dt^2 - a(t)d\mathbf{x}^2 \quad . \tag{45.1}$$

After proving the case of a toy model, they proceed to prove the general case similarly. The proof for the toy model begins with the differential element of the null interval in the space described by (45.1):

$$d\lambda \propto a(t)dt \quad . \tag{45.2}$$

This may be normalized as

$$d\lambda \propto \frac{a(t)}{a(t_f)}dt \quad , \tag{45.3}$$

for some reference time t_f. Using the standard definition of the Hubble parameter

$$H = \frac{\dot{a}}{a} \quad , \tag{45.4}$$

the authors integrate H from an early time to t_f:

$$\begin{aligned}
\int_{\lambda(t_i)}^{\lambda(t_f)} H(\lambda)d\lambda &= \int_{t_i}^{t_f} \frac{\dot{a}(\lambda)}{a(\lambda)}\,d\lambda \\
&= \int_{t_i}^{t_f} \frac{\dot{a}(t)}{a(t)}\,\frac{a(t)}{a(t_f)}dt \\
&= \int_{t_i}^{t_f} \frac{d}{dt}\frac{a(t)}{a(t_f)}dt \\
&= \int_{a(t_i)}^{a(t_f)} \frac{da}{a(t_f)} \\
&= \left(1 - \frac{a(t_i)}{a(t_f)}\right) \leq 1 \quad .
\end{aligned} \tag{45.5}$$

The authors then define an average Hubble parameter H_{av} over the affine parameter λ:

$$H_{\text{av}} = \frac{1}{\lambda(t_f) - \lambda(t_i)} \int_{\lambda(t_i)}^{\lambda(t_f)} H(\lambda)d\lambda \leq \frac{1}{\lambda(t_f) - \lambda(t_i)} \tag{45.6}$$

According to the metric in (45.1), the universe has always been expanding, and it follows that $H_{\text{av}} > 0$. Since (45.6) shows that the average has to be less than or equal to the given fraction, the affine parameter is constrained to some finite length. H_{av} is greater than zero, but the fraction goes to zero as $\lambda(t_i) \to -\infty$. Thus, $H_{\text{av}} > 0$ requires that the time interval parameterized by λ cannot extend infinitely far into the past.

The BGV theorem does not cover the case of χ^4 extending infinitely far into the chirological past, so the theorem does not disrupt the presumed infinite extent of the MCM cosmological lattice. While the MCM does not necessarily depend, at this point, on a cyclic cosmology model in which timelike chronological geodesics must extend infinitely far down the past light cone, the introduction of a reversed time arrow at the past spacetime boundary implied by (45.6) may provide a workaround for the implication of the BGV theorem. Rather than forcing past incompleteness, the BGV theorem might be shown to force sign-alternating piecewise structure onto the affine parameterization of geodesics longer than some scale.

46 The Ehrenfest Paradox

The Ehrenfest paradox [179] pertains to a spinning disc whose outer edge moves at relativistic speeds. The radius of the disc is always perpendicular to the motion of the disc's elements and should not be affected by special relativistic length contraction. The circumference of the disc, however, is parallel to the tangential velocity of the disc's elements and must be affected by length contraction. As a consequence, the ratio π between the non-spinning disc's radius R_0 and circumference C_0 cannot be the ratio of R_0 to the spinning disc's circumference C. An extensive analysis of this problem appears in [180] wherein Grøn emphasizes a historical preoccupation with the elastic properties of a hypothetical disc as well as the feasibility of making properly comoving measurements to observe the ratio $C/R \neq 2\pi$. Without regard to the material properties of a physical disc or the possibility of any specific experiment, the Ehrenfest paradox is a fascinating problem in pure geometry.

Framing this process in the MCM, one would examine the case in which π is factored out of Lorentz contraction in a rotating reference frame as

$$\big|\psi;\hat{\pi}\big\rangle \quad\longrightarrow\quad \underbrace{\big|\psi\big\rangle}_{\text{relativistic}} \times \underbrace{\hat{\pi}}_{\text{abstract}} \quad . \tag{46.1}$$

This possibility is directly falsifiable. If the mechanism is not immediately determined to be infeasible, one would compare any results to known gravitational anomalies such as the Pioneer anomaly [181].[1] Small deviations from the predicted orbit might be assigned to a non-relativistic $\hat{\pi}$.

[1] The current reliance on finite element analysis to diagnose the Pioneer anomaly as a heat issue [182] is unsatisfying to this writer. The small effect obtained in this analysis is as likely to be an artifact of the choice of finite elements as it is to be a real effect.

47 The Ford Paradox

In [183], Locklin writes the following about the Ford paradox [184, 185].

> "If quantum mechanics is the ultimate theory of the universe: where do the long strings of random bits come from in a classically chaotic system? Since people believe that QM is the ultimate law of the universe, somehow we must be able to recover all of classical physics from quantum mechanics. This includes information generating systems like the paths of chaotic orbits. If we can't derive such chaotic orbits from a QM model, that indicates that QM might not be the ultimate law of nature. Either that, or our understanding of QM is incomplete. Is there a point where the fuzzy QM picture turn into the classical bit generating picture? If so, what does it look like in the transition?
>
> "I've had physicists tell me that this is 'trivial,' and that the 'correspondence principle' handles this case. The problem is, classically chaotic systems egregiously violate the correspondence principle. Classically chaotic systems generate information over time. Quantum mechanical systems are completely defined by stationary periodic orbits. To say the 'correspondence principle handles this' is to merely assert that we'll always get the correct answer, when, in fact, there are two different answers. The Ford paradox is asking the question: if QM is the ultimate theory of nature, where do the long bit strings in a classically chaotic dynamical system come from? How is the classical chaotic manifold constructed from quantum mechanical fundamentals?"

While the above pertains to the chaotic double pendulum, one might ask where large scale turbulence in the universe comes from if the big bang was a *quantum* nucleation event. If quantum mechanics were the supreme law in the evolution of the universe, it would follow that the universe could have only become a perfectly symmetric crystal devoid of the information needed to characterize turbulent macro-scale structures. Since QM is an information-conserving theory, the universe at late times would not contain more information than was in the initial qubit.

Locklin also writes the following [183].

> "This may seem subtle, but according to quantum mechanics, the 'motion' is completely defined by periodic orbits. There are no chaotic orbits in quantum mechanics. In other words, you have a small set of periodic orbits which completely define the quantum system. If the orbits are all periodic,

> there is less information content than orbits which are chaotic. If this sort of thing is true in general, it indicates that classical physics could be a more fundamental theory than quantum mechanics."

While the Ford paradox is little regarded by the physics orthodoxy, it was instrumental in this writer's thinking during the formation of the MCM. Fundamentally, the increase of entropy in real systems requires that total information is not conserved. This is not possible in the framework of quantum mechanics. The introduction of irrational numbers to the scale factor associated with non-unitary $\hat{M}^3$ is proposed in part to generate the anomalous (but required) long strings of random bits needed to describe simple experiments and, eventually, large scale turbulence in the universe, e.g.: crashing ocean waves, terrestrial clouds, astrophysical nebulae, etc. For instance, new information might enter a theory when the arguments of sines and cosines periodic in 2π are rescaled as $n\pi x \to n\pi\Phi x'$. Cases of this paradox must be surveyed and analyzed in the modified framework of quantum mechanics dependent on $\hat{M}^3$.

48 Intrinsic Periodicity

A program of Dolce, e.g.: [186–189], pertains to the periodicity intrinsic to quantum states through the de Broglie wavelength. Dolce endeavors to associate this manifestation of wave-particle duality with more fundamental periodicities in time and space. Given the MCM particle scheme in which fundamental particles are themselves quanta of spacetime more so than they are the "isolated energy parcels" described by de Broglie [190], a survey of Dolce's program is likely to yield results, methods, and language appropriate for use in the MCM. Particularly, Dolce has considered the case of a virtual extra dimension in [188]. The MCM's fifth *abstract* dimension may be more appropriately classified as a canonically virtual dimension than a real spacelike or timelike one. The distinctions of such dimensions must be analyzed and incorporated into the MCM to the extent that they are useful.

49 Non-Local Hidden Variables

Bell's theorem is said to preclude the existence of local hidden variables in quantum theory. 'T Hooft states Bell's theorem as follows [191].

> "No physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics."

A hypothetical MCM workaround for this theorem was meant to allow χ^4 as a local hidden variable [192]. However, the mechanism relies on several unresolved *if* statements, and subsequent analysis shows that χ^4 may be better described as a non-local hidden variable than a local one.[1] Therefore, it is required to establish χ^4 as a non-local variable or to manufacture a rigorous refutation of Bell's theorem.

50 ΛCDM Cosmology

Energy equations are rarely sufficient to determine unique solutions in cosmology. In general, equations of state are required before one may form a system of N cosmology equations in N unknowns. In this research program, we have not considered pressures and densities at all. We have only loosely alluded to the associations of AdS and dS with positive and negative cosmological constants, but those constants are intimately linked to the thermodynamic state of a universe. ΛCDM models cover all standard thermodynamic cosmology states, so these models must be canvased for the additional constraint equations required in a mature model of cosmology, e.g.: Chapter 27 in [193] or Chapter 8 in [194].

51 The Landau–Yang Theorem

Although Particle Data Group reports each year that it has not yet been determined if the Higgslike particle decays to two photons, e.g.: [17, 27, 124, 125], it is said that spin-1 is ruled out for that particle by the Landau–Yang theorem *because the particle has been observed to decay to two photons*. In either case, a review of this theorem [195,196] and its foundation in spin-statistics is in order. If it is eventually determined that the Higgslike particle decays to two photons, might there be a workaround for the implication of the Landau–Yang theorem that spin-1 is consequently forbidden? The foundations of this theorem must be analyzed to determine whether or not new principles inherent to the MCM might introduce corner cases which have not been previously considered.

52 A Clopen Universe

While it is an open question whether or not the physical universe is topologically flat, all data indicates that the deviation from topological flatness on cosmological scales is very small if it exists at all. However, the 4D de Sitter and anti-de Sitter slices of

[1] The information and correlations of local variables are limited by the speed of light.

the MCM unit cell have positive and negative curvature respectively corresponding to topological open- and closedness. The $\mathcal{H}$-brane can smoothly sew together the geometries of the low curvature limits of the slices of $\Sigma^{\pm}$, but the topology of $\mathcal{H}$ cannot smoothly merge the incompatible topologies. An eccentric topology beyond closed and open is the *clopen* topology. Something is said to be clopen if it is both open and closed. One would conduct a survey of the properties of clopen spaces with the goal of better understanding connections between $\Sigma^{\pm}$. One would seek to develop cosmological observables related to topological clopenness in the $\mathcal{H}$-brane.

53 The Galactic Rotation Anomaly

To a hammer, every problem looks like a nail. This is the main reason why particle physicists insist on solving the galactic rotation anomaly with particles despite a mountain of non-confirming evidence. So many experiments have failed to detect even the slightest hint of dark matter that modern dark matter models are hopelessly contrived and unrealistic. At the heart of this active research area is a large federal budget for grants related to dark matter investigations. Without that, most reasonable scientists would have given up on the particle theory of dark matter by now. Dozens or hundreds of experiments have failed to find any evidence.

Underlying a hypothetical, undetectable or nearly undetectable, novel form of matter called dark matter is the galactic rotation anomaly. The tangential velocities of stars on the outer rims of spiral galaxies are too large for those stars to be held within their galaxies by the gravity of the visible matter. Thus, it was originally speculated that there must be some additional, invisible matter called *dark matter* responsible for the gravitational binding of the fast-moving stars on the rim. In analogy, if one pours sand on a dinner plate and begins to spin the plate, the sand will begin to fly off from the outer edge of the plate once the velocity exceeds what can be offset by the sand's friction with the plate. Likewise, certain stars should be thrown out of their galaxies, but that is not what is observed. The anomaly by which stars remain on the spinning plate is very real, but, in light of overwhelming experimental evidence, there is little to no good reason to suppose that a new form of particulate *dark* matter generates the required gravitation. As stated above, it is this writer's opinion that all or nearly all interest in such models pertains to a large pool of funding for what is effectively a dead research area. Such research is kept alive because there is always a risk that funds will be diverted to sociology rather than other problems in physics.

As an alternative to a novel, dark form of matter, one would attempt to account for the anomalous gravitation by exploring galactic geometries other than the simplest ones which can be extracted from 2D astronomical data recorded by telescopes. For instance, one would explore the gravitational energy of 4D galactic geometries rather than only 3D geometries. The MCM suggests dark energy as a cosmological effect when the universe gravitates toward another universe (or itself) in the future (Section 7), and one would also explore localized galactic manifestations of such effects. Might the mass of the galaxy in the future gravitate with visible matter on our past light cone? Might other, more exotic galactic configurations be consistent with 2D astronomical data and also able to account for the anomalous gravitation? Matter hidden from our telescopes by exotic geometries might be labeled "dark" without reference to new particles absent from the standard model.

An anomalous correlation of supermassive black hole masses with the masses of their host galaxies is further evidence that one ought to explore alternative models of galactic physics. Present models predict that a black hole in the center of a galaxy should not know anything about the mass of the galaxy itself, but observations show that heavier galaxies tend to host heavier supermassive black holes [197, 198]. Rather than posing dark matter for one anomaly and another theory for the other, one would seek a new model of galactic physics in which to resolve both issues.

54 Neutrino Helicity

The standard model predicts that all neutrinos should exist in left-handed helicity states. However, observed neutrino flavor oscillations [199–201] require that neutrinos must exist in left- and right-handed helicities. This major deficiency of the standard model's wrong prediction should be examined in the context of the MCM particle scheme. Relative to conserved parity for strong and EM interactions, one would seek to identify a mechanism in the structure of the unit cell for parity non-conservation in weak interactions. One would develop MCM cases for Dirac and Majorana neutrinos and study the standard model case in which right-helical Dirac neutrinos cannot interact via the weak interaction. One would attempt to attribute the standard model prediction to inherent directionality in $\hat{M}^3$.

In one further portion of neutrino physics, one might revisit the MCM particle scheme (Section 0.2) in which neutrinos are differentiated from their charged lepton partners by the handedness of a $\{x^0, x^i, \chi^4\}$ coordinate system. As mentioned in Section 1.9.1, the subsequent introduction of $\widehat{\infty}$ may allow us to distinguish charged lepton/neutrino pairs between coordinate triads located at either of $\widehat{0}$ of $\widehat{\infty}$ rather than between right- and left-handed ones. Such an arrangement might lend itself to the large mass ratios among charged and neutral leptons, and to a longstanding prediction for massless neutrinos which was ultimately rejected on the basis of neutrino oscillations. In an intuitive way, one would associate zero mass with $\widehat{0}$ or $\widehat{\infty}$, and then develop an allowance for a small non-zero mass pertaining to the neighborhood of infinity. To avoid similar mass ratios among generations of quarks paired according to $\widehat{0}$ and $\widehat{\infty}$, one would appeal to the piecewise structure of χ^4 relative to x^0.

55 Local Gauge Symmetry

Consider the standard model of particle physics' group theoretical structure $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. While there are variations on the standard model which are allowed, this algebraic structure seems to be enforced at the experimental level. The U(1) part describes, loosely, the oscillations of the EM field. From one point in spacetime to another, the EM field has a U(1) symmetry such that $\mathbf{E}$ is determined by $\theta\in[0,2\pi)$ and $\mathbf{B}$ is determined consequently by overall relativistic invariance. Every possible value of the EM field at a point can be obtained by applying a U(1) rotation to the value of θ at any other point. Therefore, we say that the the EM field has a U(1) phase at each point. In quantum mechanics, the U(1) gauge symmetry allows us to make changes like $\psi\to e^{i\lambda}\psi$ as long we make corresponding *gauge transformations* elsewhere in the theory. In this case, the U(1) circle group describes the 2π radian periodicity in the function $e^{i\lambda}=e^{i\lambda+2\pi}$. The SU(2) weak theory is slightly more complicated. It adds a 2-sphere of coordinate freedom to each point in spacetime and the U(1) part of the $\mathrm{SU}(2)\times\mathrm{U}(1)$ electroweak theory pertains to *hypercharge* rather than electric charge. The SU(3) strong force adds a 3-sphere to each point. The strong force at any point can be obtained by rotating three QCD angles defined at any other point. These degrees of freedom assigned to the points of spacetime are called local gauge symmetries.

Following the MCM particle scheme in which gravitational manifolds are like elementary particles, one would link the dynamical quantum metric defined with four degrees of freedom to a new local QFT gauge symmetry. In other words, the internal coordinates associated with the gauge symmetries at a point on one level of aleph would be like the coordinates of a gravitational manifold on another level of aleph. This description of metrical degrees of freedom as local gauge symmetries may be useful in MCM quantum gravity which resolves quantum states as metric tensors. For example, the necessary slow variation of x^μ Cartesian coordinates in spacetime relative to an SU(4) representation of those coordinates as four angles may be germane to the hierarchy problem.

56 The Amplituhedron

In late 2013, several months after the first publication of the unit cell in [7], Arkani-Hamed and Trnka published a famously received paper describing an "amplituhedron." They write the following in [202].

> "Perturbative scattering amplitudes in gauge theories have remarkable simplicity and hidden infinite dimensional symmetries that are completely obscured in the conventional formulation of field theory using Feynman diagrams. This suggests the existence of a new understanding for scattering amplitudes where locality and unitarity do not play a central role but are derived consequences from a different starting point. In this note we provide such an understanding for $N = 4$ [*supersymmetric Yang–Mills theory (SYM)*] scattering amplitudes in the planar limit, which we identify as 'the volume' of a new mathematical object—the Amplituhedron."
>
> "[*T*]here must be a different formulation of the physics, where locality and unitarity do not play a central role, but emerge as derived features from a different starting point. A program to find a reformulation along these lines was initiated in [*two omitted Arkani-Hamed papers*], and in the context of a planar $N = 4$ SYM was pursued in [*three other omitted Arkani-Hamed papers*], leading to a new physical and mathematical understanding of scattering amplitudes [*in another omitted Arkani-Hamed paper*]."

The new object with volume is the authors' version of the unit cell. Following the program of Ashtekar [49, 57], Finkelstein [146–152], *and others*, Arkani-Hamed and Trnka appear to suggest that they were barking up the same tree as this writer, at the same time, and without delivering any hard new results of their own, at that time or subsequently.[1] As Finkelstein was so enchanted by the idea to model the big bang with a Feynman diagram (Figure 29), the 2013 comment of Arkani-Hamed and Trnka regarding post-Feynmanian understanding of scattering amplitudes suggests that they were enamored of the idea as well, and rightly so. The reader is reminded that a modestly similar likeness demonstrated by Maldacena became *the most cited paper* in the high energy particle physics literature [58] despite Maldacena not having demonstrated what new physics or precise new understanding might be gained by the AdS/CFT correspondence. The likeness between the anti-de Sitter space and a conformal field theory in one less dimension speaks for itself. Masterful scholars infer

[1] To the contrary, this writer has parlayed early thinking into the negation of the Riemann hypothesis [2, 46, 47], a formidable result of hard matériel, and made numerous other technical advances.

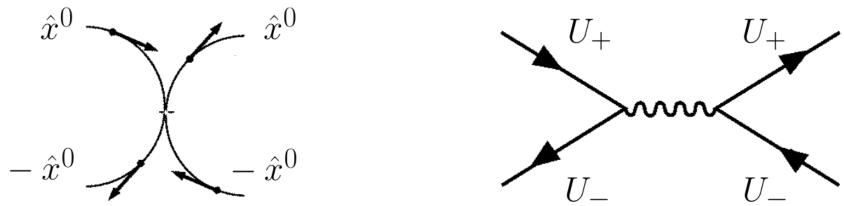

Figure 29: This figure is adapted from [39]. The original caption read, "A duality transformation between the geometric and particle pictures."

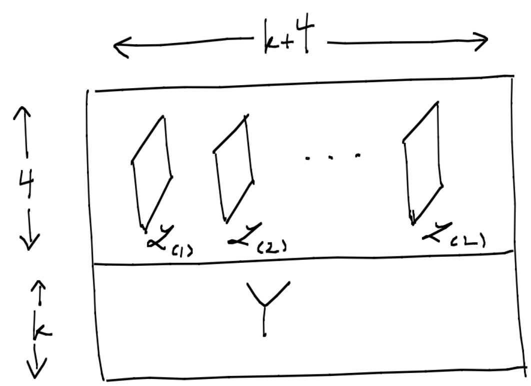

Figure 30: This figure is due to Arkani-Hamed and Trnka [202]. We suggest that this figure is intended to invoke the likeness of the MCM unit cell. The "4" written to the left apparently refers to the dimensionality of the MCM's labeled branes, and k refers to an unspecified number of embedding dimensions: the MCM's $\chi^4_\pm$ (and $\chi^4_\varnothing$.)

from the likeness alone that it should be important *for something*. Indeed, it is likely that far more physicists were excited by the MCM particle-spacetime duality than were this writer, Arkani-Hamed, and Finkelstein, c.f.: Appendix C.

Arkani-Hamed and Trnka continue as follows.

> "[W]e can now give the full definition of the amplituhedron[.] [...] The amplituhedron lives in $G(k, k+4; L)$: the space of k planes Y in $(k+4)$ dimensions, together with L 2-planes $\mathcal{L}_i$ in the 4 dimensional complement of Y, [*as in Figure 30.*] [...] The amplituhedron $A_{n,k,L}(Z)$ is the subspace of $G(k, k+4; L)$ consisting of all $\mathcal{Y}$'s which are a positive linear combination of the external data[.] [...] The notion of cells, cell decomposition and canonical form can be extended to the full amplituhedron. A cell Γ is associated with a set of positive coordinates $\alpha^\Gamma = (\alpha^\Gamma_1, \ldots, \alpha^\Gamma_{4(k+L)})$, rational in the $\mathcal{C}$, such that for α's positive, $\mathcal{C}(\alpha) = (D_{(i)}(\alpha), C(\alpha))$ is in $G_+(k, n; L)$. A cell

> decomposition is a collection T of non-intersecting cells Γ whose images under $\mathcal{Y}=\mathcal{C}\cdot Z$ cover the entire amplituhedron."

In the opinion of this writer, Arkani-Hamed examined the unit cell and envisioned Figure 30 as his own alternative ideation. After that, all of the language about, "generalizing the notion of the inside of a triangle in a plane," [202] (not excerpted) was reverse engineered to develop a context for Figure 30 without being forced to acknowledge the MCM or ideation from this writer. Indeed, Arkani-Hamed and Trnka appear to go much farther than their peers in generating a paper trail to suggest their own parallel ideation. To wit, after sufficiently *generalizing the inside of a triangle*, Arkani-Hamed and Trnka state the above in which a decomposable Γ takes the place of $\Sigma^+\cup\Sigma^-$. The "entire amplituhedron" is presumably the analogue of the MCM bulk cosmological lattice. The authors' following claim (below) to be motivated by the idea of the area of dual polygon is more evidence that the main goal of their paper was to rephrase the MCM's dual $\Sigma^\pm$ structure in the context of their own legitimate research, and a smattering of nonsense given as excuse.

> "While cell decompositions of the amplituhedron are geometrically interesting in their own right, from the point of view of physics, we need them only as a stepping stone to determining the form $\Omega_{n,k,L}$. This form was motivated by the idea of the area of a (dual) polygon."

It seems unlikely to this writer that hot-shot particle physicists at the Institute for Advanced Study would have found spontaneous and profound inspiration in a polygon's dual having an area. In the opinion of this writer, the amplituhedron is more evidence of the great worth of this writer's ideas being immediately apparent to everyone exposed to them, barring those less expert in physics. As with pseudo-plagiaristic rewrites lacking proper acknowledgment of ideation in other sections, one would survey work pertaining to the amplituhedron to determine if any new insights were had and whether or not any of them might be useful for developing the MCM.

57 Time Crystals

A time crystal is a physical system which is periodic in time. The MCM unit cell is an example of a time crystal, and, indeed, the term "unit cell" is taken from the physics of crystalline solids. Wilczek's and Shapere's time crystal papers [51, 203] appeared on arXiv in February 2012, a year before the unit cell was published in [7]. Therefore, we will consider the classical time crystal paper authored by Shapere and Wilczek, and the quantum time crystal paper authored by Wilczek alone, as response papers[1] to the time periodicity attributed to $\hat{M}^3$ near the end of 2011 [30, 39]. To Wilczek's credit, he seems only to analyze the possibilities related to $\hat{M}^3$ without attempting to fabricate a parallel construction. He writes the following [51].

> "Symmetry and its spontaneous breaking is a central theme in modern physics. Perhaps no symmetry is more fundamental than time translation symmetry, since time translation symmetry underlies both the reproducibility of experience and, within the standard dynamical frameworks, the conservation of energy. So it is natural to consider the question, whether time translation symmetry might be spontaneously broken in a closed quantum mechanical system. That is the question we will consider, and answer affirmatively, here. Here we are considering the possibility of time crystals, analogous to ordinary crystals in space. They represent spontaneous emergence of a clock within a time-invariant dynamical system. [...]
>
> "Several considerations might seem to make the possibility of quantum time crystals implausible. The Heisenberg equation of motion for an operator with no intrinsic time dependence reads
>
> $$\left\langle \Psi \middle| \dot{\mathcal{O}} \middle| \Psi \right\rangle = i \left\langle \Psi \middle| [H, \mathcal{O}] \middle| \Psi \right\rangle \quad \underset{\Psi = \Psi_E}{\rightarrow} \quad 0 \ , \tag{57.1}$$
>
> where the last step applies to any eigenstate Ψ_E of H. This seems to preclude the possibility of an order parameter that could indicate the spontaneous breaking of infinitesimal time translation symmetry. Also, the very concept of 'ground state' implies state of lowest energy; but in any state of definite energy (it seems) the Hamiltonian must act trivially. Finally, a system with spontaneous breaking of time translation symmetry in its ground state must have some sort of motion in its ground state, and is therefore perilously close to fitting the definition of a perpetual motion machine."

In [203], Shapere and Wilczek state the following.

> "When a physical solution of a set of equations displays less symmetry than the equations themselves, we say the symmetry is spontaneously broken by that solution."

[1] These papers are very good, concise, and readable. The mathematical analysis therein is greatly contrasts the mathematical content of [202] (Section 56) in that it cannot be considered math salad by any stretch of the imagination. Wilczek's and Shapere's math is math as it is meant to be. Both papers are models of good work in fundamental physics, excepting the likely lack of a due acknowledgment.

One understands that the three-fold process for $\hat{M}^3$ gives a context in which general classes of solutions might display far more complex behavior than is immediately apparent in $\mathcal{H}\to\Omega\to\mathcal{A}\to\mathcal{H}$. Particularly, Wilczek's attention to violation of time reversal symmetry in [51] places the content of his paper later than the postulation of $\hat{M}^3$ [30,39] but earlier than the postulation of the unit cell [7]. In 2012, $\hat{M}^3$ would require violation of time translation symmetry for a particle at rest, presumably in the ground state, to move in a periodic motion among $\{\mathcal{A},\mathcal{H},\Omega\}$. This reflects the $t_0 \to t_{\text{max}} \to t_{\text{min}} \to t_0$ convention of [30, 39], but the subsequent introduction of the chirological time spanning the unit cell in 2013 [7] sidesteps the main thrust of Wilczek's paper. Still, time crystals were affirmatively observed in 2016 [204], and we have very much designed the unit cell to function like a clock: the ticking of $\hat{M}^3$ from one observation to the next marks the time. The constant cycling among the branes of the unit cell during repeated observations of a system in its ground state is very much like perpetual motion. However, abstract perpetual motion in the bulk of the unit cell may not be prohibited as is physical perpetual motion in $\mathcal{H}$.

The point raised by (57.1) is that any order parameter, call it the expectation value $\langle\mathcal{O}\rangle$, which might indicate spontaneous symmetry breaking by a transition from trivial into more complicated behavior is forbidden in the ground state due to the expectation value of $\dot{\mathcal{O}}$ being equal to zero. By imposing the overall symmetry condition (periodicity) in the unit cell in the χ^4 direction, we do not impose any new symmetries on the x^μ part of a Hamiltonian, so the extent to which Wilczek's work on symmetry breaking is in scope for the current iteration of the MCM must be evaluated.

Following the wrap-the-x^0-axis-around-cylinder program of the earliest incarnation of the MCM [31], Wilczek cites a ring particle as an exception to his previous comment regarding the preclusion of an order parameter (not excerpted.) For an angular coordinate ϕ and a Lagrangian

$$L = \frac{1}{2}\dot{\phi}^2 + \alpha\phi \quad , \tag{57.2}$$

Wilczek makes the following remarks.

> "If α is not an integer, we will have
>
> $$\big\langle l_0\big|\dot{\phi}\big|l_0\big\rangle = l_0 - \alpha \neq 0 \quad . \tag{57.3}$$
>
> The case when α is half an odd integer requires special consideration. In that case we will have two distinct states $\big|\alpha\pm\frac{1}{2}\big\rangle$ with the minimum energy."

The case of α being half an odd integer leading to two ground states deserves closer study because it is qualitatively suggestive of the $\Sigma^{\pm}$ structure as well as the structure of the fundamental fermions in the MCM particle scheme. Wilczek makes other remarks regarding multiple ring particle wavefunctions behaving as Cooper pairs. The utility of this language must also be evaluated for descriptions of the $U_{\pm}$ universes' particle descriptions. The language of Cooper pairs is well suited to the picture of $U_{\pm}$ universes in simultaneous, isentropic cosmological bouncing.

Further hinting at underlying ideation in the MCM, Wilczek writes the following [51].

> "It is interesting to speculate that a (considerably) more elaborate quantum-mechanical system, whose states could be interpreted as collections of qubits, might be engineered to traverse, in its ground configuration, a programmed landscape of structured states in Hilbert space over time. [...] The a.c. Josephson effect is a semi-macroscopic oscillatory phenomenon related in spirit to time crystallization. It requires, however, a voltage difference that must be sustained externally."

One speculates that the more elaborate system is the one described by $\mathcal{H}\to\Omega\to\mathcal{A}\to\mathcal{H}$ which became the unit cell and its accoutrements detailed in Section 1. The Josephson junction is the main experimental protocol for measuring the fine structure constant, and a comment of Shapere and Wilczek in [203] about higher powers of velocities appearing naturally seems to refer to the third derivative application for $\alpha^{-1}_{\text{MCM}} = 2\pi + (\Phi\pi)^3$. Another comment in [203] about converting space derivatives into time derivatives seems to refer to the statement

$$\hat{\mathcal{U}} \propto \partial_x \quad , \qquad \text{and} \qquad \hat{M} \propto \partial_t \quad , \tag{57.4}$$

which was the main proposal for a new mathematical method before χ^4 was introduced in [7]. Therefore, one questions the words of Ledesma-Aguilar [205]:

> "Back in 2012, Wilczek came up with a tantalizing idea. He wondered if, in the same way that a crystal breaks symmetry in space, it would be possible to create a crystal breaking an equivalent symmetry in time. This was the first time the idea of a time crystal was theorized."

As stated above, it is to Wilczek's credit that he has not fabricated some fictitious path toward the idea of time crystals. He simply writes about the idea. Despite Wilczek not making any claim toward independent ideation, however, his non-citation to the MCM facilitates the above easy and natural misconception. Similarly, Weinstein never claimed to have formulated the theory of "Geometric Unity" (to the knowledge of this writer) [206], but his failure to attribute the original ideation to Tooker facilitated a profound and rampant misconception.

It remains true that time crystals were observed in 2016 [204], and the topic is an active research area in fundamental physics today. The main results in the field [204, 207–209] must be surveyed and evaluated for applications in the MCM. The main question will be whether the current structure of the MCM unit cell requires the breaking of time translation symmetry and, if it does, whether or not χ^4 should be useful as an order parameter characterizing the broken symmetry.

58 Cellular Automata

'T Hooft has written a book called *The Cellular Automaton Interpretation of Quantum Mechanics* [210] which begins as follows.

> "This book is about a theory, and about an interpretation. The theory, as it stands, is highly speculative. It is born out of dissatisfaction with the existing explanations of a well-established fact. The fact is that our universe appears to be controlled by the laws of quantum mechanics. Quantum mechanics looks weird, but nevertheless it provides for a very solid basis for doing calculations of all sorts that explain the peculiarities of the atomic and sub-atomic world. The theory developed in this book starts from assumptions that, at first sight, seem to be natural and straightforward, and we think they can be very well defended.
>
> "Regardless whether the theory is completely right, partly right, or dead wrong, one may be inspired by the way it looks at quantum mechanics. We are assuming the existence of a definite 'reality' underlying quantum mechanical descriptions. The assumption that this reality exists leads to a rather down-to-earth interpretation of what quantum mechanical calculations are telling us. The interpretation works beautifully and seems to remove several of the difficulties encountered in other descriptions of how one might interpret the measurements and their findings. We propose this interpretation that, in our eyes, is superior to other existing dogmas."

'T Hooft expresses the same interest in going beyond quantum mechanics which motivates the MCM. Fortunately, one finds evidence in the literature [211, 212] that 't Hooft was pursuing such ideas before the MCM was constructed. Indeed, 't Hooft's references to the problem of information loss (Section 47) in [210, 211] paint a picture of his interests being well aligned with those of this writer. Regarding the specifics of his model, he writes the following.

> "A *cellular* automaton is an automaton where the data are imagined to form a discrete, d -dimensional lattice, in an $n = d + 1$ dimensional space-time. The elements of the lattice are called 'cells', and each cell can contain a limited amount of information. The data $Q(\vec{x}, t)$ in each cell $(\vec{x}, t)$ could be represented by an integer, or a set of integers, possibly but not necessarily limited by a maximal value $\mathbb{N}$. An evolution law prescribes the values of the cells at time $t + 1$ if the values at time t (or t and $t - 1$) are given. Typically, the evolution law for the data in a cell at the space-time position
>
> $$(\vec{x}, t) \quad , \quad \vec{x} = (x^1, x^2, ...x^d) \quad , \quad x^i, t \in \mathbb{Z} \quad , \tag{58.1}$$
>
> will only depend on the data in neighbouring cells at $(\vec{x}', t-1)$ and possibly those at $(\vec{x}', t - 2)$."

The MCM exceeds 't Hooft's model in that the presence of a third derivative will require data at least as far back as $(\vec{x}', t - 3)$ due to the backward difference approximation of the third derivative. Retrocausality suggests data as far forward as $(\vec{x}', t + 3)$ as well, but those would make for tricky calculations since those data do not exist at time t. Overall, 't Hooft's cellular automaton is quite like the MCM. His insights on such physics are likely to provide clarifications on the structure of the MCM cosmological lattice. 'T Hooft's lattice description raises an interesting question, actually, in that we have not decided if all non-local lattice sites contribute to local physics (as terms in a Laurent series for example), or if we might restrict interactions with $|\psi; \hat{e}^k_\mu\rangle$ to $|\psi'_\lambda; \hat{e}^{k\pm n}_\nu\rangle$ for $0 \leq n \leq 3$. For comparison, the derivation of 10^{-4}m as the characteristic length scale for new effects referenced only one higher $\hat{\pi}$-site and one lower one (Section 15).

'T Hooft explains the kernel of his model as follows [210].

> "The price we do pay seems to be a modest one, but it needs to be mentioned: we have to select a very special set of mutually orthogonal states in Hilbert space that are endowed with the status of being 'real'. This set consists of the states the universe can 'really' be in. At all times, the universe chooses one of these states to be in, with probability 1, while all others carry probability 0. We call these states *ontological states*, and they form a special basis for Hilbert space, the ***ontological basis*** [*emphasis added*].

One could say that this is just wording, so this price we pay is affordable, but we will assume this very special basis to have special properties. What this does imply is that the quantum theories we end up with all form a very special subset of all quantum theories. This then, could lead to new physics, which is why we believe our approach will warrant attention: eventually, our aim is not just a reinterpretation of quantum mechanics, but the discovery of new tools for model building."

Unfortunately, 't Hooft's book trails this writer's "Ontological Physics" [70] by about five months. The main topic of that paper was that an observer has no way to tell if his *basis* for quantum mechanics is on one level of aleph or another.[1] One notes that the extensive reference to Bell's theorem in 't Hooft's introduction follows "On Bell's Inequality" [192] by that same five months. One might ask whether the comments in 't Hooft's introduction regarding the theory being "completely right, partly right, or dead wrong" refer respectively to the MCM model of particles [6], the framework for levels of aleph [70], and the proposed workaround for Bell's inequality [192], all of which were published in 2013.[2] Whatever the circumstances, 't Hooft's program should be investigated. The following definitions from [210] are of particular interest.

"We plan to distinguish three types of operators:

(I) *beables*: these denote a property of the ontological states, so that beables are diagonal in the ontological basis $\{|A\rangle, |B\rangle, ...\}$ of Hilbert space:

$$\mathcal{O}_{\mathrm{op}}|A\rangle = \mathcal{O}(A)|A\rangle \quad , \qquad \text{(beable)} \quad . \tag{58.2}$$

(II) *changeables*: operators that replace an ontological state by another ontological state, such as a permutation operator:

$$\mathcal{O}_{\mathrm{op}}|A\rangle = |B\rangle \quad , \qquad \text{(changeable)} \quad . \tag{58.3}$$

These operators act as pure permutations.

[1] The first verbatim appearance of the phrase "ontological basis" in the MCM is traced to 2015 [160]. It is possible that this writer had already had a look at 't Hooft's book when coining the term. However, the ontological basis cited in [160] was a clear continuation of the work regarding ontology and bases begun in [70]. The verbiage was probably found independently.

[2] As mentioned in Section 49, a number of unresolved conditions in the proposed workaround for Bell's theorem [192], and further evidence for the non-locality of χ^4 suggest that the workaround may be infeasible, or "dead wrong."

(III) *superimposables*: these map ontological states onto superpositions of ontological states:

$$\mathcal{O}_{\text{op}}|A\rangle = \lambda_1|A\rangle + \lambda_2|B\rangle + ... \quad ." \tag{58.4}$$

To specify that which is of most interest, the time arrow states we have built from

$$\widehat{\text{MCM}}\big|\text{bounce}\big\rangle = \big|t_+\big\rangle + \big|t_-\big\rangle \quad , \tag{58.5}$$

cast $\widehat{\text{MCM}}$ as a superimposable operator.

59 Feynman's Division of the Time Interval

In his spacetime approach to non-relativistic quantum mechanics, Feynman identifies a shortcoming of his framework [67]. After showing that the action

$$S[x(t)] = \int dt\, L(x(t), \dot{x}(t)) \quad , \tag{59.1}$$

can be divided into small steps along any path as

$$S = \sum S(x_i, x_{i-1}) \quad , \tag{59.2}$$

Feynman writes the following [67].

> "Actually, the sum in [*(59.2)*], even for finite [*time steps*] is infinite and hence meaningless (because of the infinite extent of time). This reflects a further incompleteness of the postulates. We shall have to restrict ourselves to a finite, but arbitrarily long, time interval."

First, we have suggested that quantum equations of motion should differ from their classical counterparts when the action along a path is maximized rather than minimized (Section 1.5). Therefore, one might reformulate Feynman's framework in that picture so as to avoid a *problem* of divergent action. Under fractional distance analysis, the sum in (59.2) will be proportional to $\widehat{\infty}$. One might also induce the superposition time $|t_\star\rangle = |t_+\rangle + |t_-\rangle$ so that the integral over t in (59.1) yields one negative action increment for every positive one so that the sum of small increments should converge in the neighborhood of the origin. As the action of two different paths through time, the corresponding equations of motion would describe two systems, one of which might be taken as Feynman's. One might develop a system for superpositions of motion following from the superposition of time. Ultimately, this would be linked to the metric's definition as a sum of contributions from $\Sigma^{\pm}$.

60 Path Integrals

The path integral measure

$$\int Dq(t) \equiv \lim_{N\to\infty} \left(\frac{-im}{2\pi\delta t}\right)^{\frac{N}{2}} \left(\prod_{k=1}^{N-1} \int dq_k\right) \quad , \tag{60.1}$$

represents an infinite-dimensional integral whose place in physics is not supported by the highest level of mathematical rigor,[1] and yet it is used to compute amplitudes in the form

$$\langle\psi_2\big|e^{-i\hat{H}T}\big|\psi_1\rangle = \int Dq(t)\, \exp\left\{i\int_0^t dt'\, \frac{1}{2}m\dot{q}_k^2\right\} \quad . \tag{60.2}$$

The integral over the oscillating complex exponential function in (60.2) cannot be guaranteed to converge, so, in general, one substitutes a complex integration variable $-it'$ for the real variable t' to obtain an exponentially damped integral which is known to converge. Since substitutions of the form $\mathbb{R}\to\mathbb{C}$ are still non-rigorous (under a certain standard of rigor), one would seek to employ the many methods for complex time listed in the previous sections to reformulate the path integral in an absolutely convergent form from physical considerations alone without the need to introduce an additional analytical step $t\to -it$, commonly called Wick rotation. The desire to obtain an effective Wick rotation for (60.2) would guide the choice of phase convention among the various steps of $\hat{M}^3$.

61 The Golden Ratio in Black Holes

The factor of 2π inherent to

$$\hat{M}^3\big|\psi,\hat{\pi}^0\rangle = 2\pi\Phi\big|\psi;\hat{\pi}^1\rangle \quad , \tag{61.1}$$

is ordinary in physics, but the coefficient's proportionality to Φ is eccentric. The golden ratio is not encountered very often in physics. One of the few places where it may be found is in the thermodynamics of black holes [213–217].[2] Since we expect to

[1] After an issue with path integral was chosen for inclusion among the problems of this book, the problem itself was wrongly identified in an earlier version of this document. Excessive haste in the drive to complete the manuscript resulted in a blunder identifying the problem and its place in the Dirac formalism. This error stands corrected.

[2] In [218], Xu and Zhong report obtaining Φ in QM. A reference therein to El-Naschie's E-infinity theory [219] puts the result of [218] in scope for the MCM because α_{MCM} was first obtained in 2011 following the program of

accrue powers of Φ in the MCM by successive transmissions through a black hole at each $\varnothing$-brane separating successive levels of aleph, a study of the black hole context for Φ is in order. In [213, 214], Davies has shown that the specific heat of a black hole transitions from negative to positive when a certain ratio exceeds the golden ratio (under certain conditions.) The transition between positive and negative specific heat evokes the picture of changing time arrow direction at the Ω-brane where $\chi_+^4 = \Phi$. Furthermore, Cruz, Olivares, and Villanueva have found that Φ is associated with the turning points of orbits around certain black holes [215]. Thus, a deep study of black holes and their thermodynamics is in order.

62 Rydberg States

Rydberg states are highly excited atomic bound states near the ionization energy. One of the least understood areas of atomic physics regards the structure of the Hamiltonian near the ionization energy. Below it, the Hamiltonian is represented as a diagonal matrix with a countable infinity of energy eigenvalues written as its diagonal entries. Beyond countable infinity, however, the infinite discrete bound states $|n\rangle$ give way to a continuum of free particle states which cannot be enumerated with integers. The Hamiltonian can no longer be represented as a true matrix above this energy, but quantum theory provides little to no guidance on the transition from matrix-valued Hamiltonians to ones with continuous analogues of rows and columns.

Numbers in the neighborhood of infinity are well suited to the study of the transition from bound states to free particle states at the ionization energy. One might limit the number of states at the Planck scale, but the mathematical structure of the theory is such that for any $n, m \in \mathbb{N}$ with $m > n$, there are an infinite number of E_m such that $E_n < E_m < E_{\text{ion}}$. With n confined to the neighborhood of the origin, it is not possible, in terms of the quantum number $|n\rangle$, even to begin to approach a state in the region of anomalous transition from discrete to continuous energy eigenvalues. However, the arithmetic of $\widehat{\infty}$ should allow us to write down the highest energy bound state as $|\widehat{\infty} - 1\rangle$. In turn, this may facilitate new methods for investigating the transition from bound states to free particle states and thereby illuminate a nebulous region of quantum theory.

El-Naschie in [220].

63 Two Time Models

The MCM is much like a two time model such as those described by Bars et al. [221–225]. Bars writes the following [222].

> "The physics that is traditionally formulated in one-time-physics (1T-physics) can also be formulated in two-time-physics (2T-physics). The physical phenomena in 1T or 2T physics are not different, but the spacetime formalism used to describe them is. The 2T description involves two extra dimensions (one time and one space), is more symmetric, and makes manifest many hidden features of 1T-physics. One such hidden feature is that families of apparently different 1T-dynamical systems in d dimensions holographically describe the same 2T system in d+2 dimensions."

Similarly, the MCM adds one spacelike and one timelike dimension as $\chi^4_\pm$ to establish a system of holographic duality between boundary physics in $\mathcal{H}$ and bulk physics in $\Sigma^\pm$. Therefore, one would conduct a survey of 2T models and phrase the MCM in those terms.

64 Time and Imaginary Time

Regarding the spacelikeness and timelikeness of $\chi^4_\pm$, one would undertake considerations regarding duality between time in QFT and imaginary time in statistical mechanics. Creutz and Freedman write the following [226].

> "Feynman's path integral formulation of quantum mechanics reveals a deep connection between classical statistical mechanics and quantum theory. Indeed, in an imaginary time formalism the Feynman integral is mathematically equivalent to a partition function."

Like the duality between a Feynman diagram and the MCM model of dual universes, *and* like the AdS/CFT correspondence, this "deep connection" between the path integral and the partition function is exciting despite any clear understanding of what the connection is. Following the program to obtain the $\{-+++\}$ signature of Minkowski space from $x^0 = ict$, the $\{-+++\pm\}$ signature in $\Sigma^\pm$ implies that there must exist an imaginary phase between $\chi^4_\pm$ relative to some affine parameter. If one is like time, the other is like imaginary time. In terms of the metric signature which results from this relative phase, they are spacelike and timelike, so the imaginary time representation will give us cause to treat both of $\chi^4_\pm$ like time, regardless of the phase.

The spacelikeness of one or the other of $\chi^4_\pm$ was mentioned in previous sections as a potential impediment to equations for time-parameterized motion across the unit cell, but the imaginary time provides a convenient workaround.

The extensive body of literature detailing the connections between time in QFT and imaginary time in statistical mechanics, including $t \to -it$ Wick rotations, should provide valuable insights into the structure of the unit cell and its cosmological lattice.

65 String Theory

A survey of string theory is in order! Particularly, the new holographic duality between the surface $\mathcal{H}$ and the bulk spaces $\Sigma^\pm$ should be associated, to the degree that it is possible, with the AdS/CFT correspondence [58] whose context in string theory is well studied in a large body of literature. As a first jump into string theory for the MCM, consider Zwiebach's remarks in [227].

> "Despite the large number of particles it describes, the Standard Model is reasonably elegant and very powerful. As a complete theory of physics, however, it has two significant shortcomings. The first one is that it does not include gravity. The second one is that it has about twenty parameters that cannot be calculated within its framework. Perhaps the simplest example of such a parameter is the dimensionless (or unit-less) ratio of the mass of the muon to the mass of the electron. The value of this ratio is about 207, and it must be put into the model by hand. [...] The first sign that string theory is rather unique is that it does not have adjustable dimensionless parameters. As we mentioned before, the Standard Model of particle physics has about twenty parameters that must be adjusted to some precise values. A theory with adjustable dimensionless parameters is not really unique. When the parameters are set to different values one obtains different theories with potentially different predictions. String theory has one dimensionful parameter, the string length ℓ_s. Its value can be roughly imagined as the typical size of strings."

One would begin a foray into the exciting field of string theory by supposing that the lengths of strings or the dimensionless ratios of strings' lengths should be proportional to the numbers in the ontological basis.[1] The S-, T-, and U-dualities of string theory should be contextualized as dualities inherent to the unit cell and the

[1] In a video lecture series on string theory, Susskind supposes that there might exist a string of length π before *greatly demurring* regarding the origin of that idea. Unfortunately, this lecture series appears to have been deleted from the internet, and a citation cannot be offered.

cosmological lattice spanning various levels of aleph. Not strangely, the dimensionality of famous 10- and 26-dimensional string theories, and 11D M-theory, is natural in the unit cell. The two 5D theories in $\Sigma^\pm$ give ten dimensions and adding x^0 makes 11. Counting

$$\chi_+^A \quad , \quad \chi_-^A \quad , \quad x_+^\mu \quad , \quad x_-^\mu \quad , \quad x^\mu \quad , \quad \text{and} \qquad \chi_\varnothing^\mu \quad , \tag{65.1}$$

shows 26 degrees of freedom when Ω and $\mathcal{A}$ are not separated by a 5-space, i.e.: when $\chi_\varnothing \neq \chi_\varnothing^A$. One would establish a link between 10- and 26D string theories by noting that the coordinates in the bounding branes should be determined by holographic duality with the coordinates in $\Sigma^\pm$.

Sen has theorized a process in string theory called *tachyon condensation* which is likely to have applications toward the MCM [228]. The superluminal quality of tachyons is directly applicable to a model of χ^4 as a non-local variable: one whose information and correlations are not restricted by the speed of light, as in Section 49. Furthermore, the thermodynamic process of condensation should introduce an equation of state which the MCM has so far ignored. Such physics may make it possible to describe the change of basis operation between chirological and chronological time arrow states as a thermodynamic phase transition from propagation in the bulk of $\Sigma^\pm$ to confinement in $\mathcal{H}$.

66 Fast Radio Bursts

This problem resides in the MCM's early venue: cosmological phenomenology. It is suggested that fast radio bursts [229, 230] should be modeled as black hole lightning. Petroff, Hessels, and Lorimer write the following [230].

> "[*P*]ulsar surveys have led to the serendipitous discovery of fast radio bursts (FRBs.) While FRBs appear similar to the individual pulses from pulsars, their large dispersive delays suggest that they originate from far outside the Milky Way and hence are many orders-of-magnitude more luminous. While most FRBs appear to be one-off, perhaps cataclysmic events, two sources are now known to repeat and thus clearly have a longer-lived central engine. [...] With peak flux densities of approximately 1 Jy, this implied an isotropic energy of 1032 J (1039 erg) in a few milliseconds or a total power of 1035 J s^{-1} (1042 erg s^{-1}.) The implied energies of these new FRBs were within a few orders of magnitude of those estimated for prompt emission from GRBs and supernova explosions, thereby leading to theories of cataclysmic and extreme progenitor mechanisms. [...] Currently, the

> research community has no strict and standard formalism for defining an FRB, although attempts to formalize FRB classification are ongoing [...]. In practice, we identify a signal as an FRB if it matches a set of loosely defined criteria. These criteria include the pulse duration, brightness, and broadbandedness, and in particular whether the [*dispersion measure*] is larger than expected for a Galactic source."

The dynamical origin of large-scale charge distributions leading to terrestrial lightning are not understood. EM theory suggests that large-scale charge formations should not appear in the atmosphere because they would seem to neutralize themselves at small-scale. However, lightning is known to occur on a scale which is only possible given unexplained large-scale charge distributions. It is also known that lightning is a radio source. Therefore, one might suppose that the mechanism for the anomalous assembly of large-scale charge distributions between a planet and its atmosphere is also in play between a black hole and its accretion matter. The famous no-hair theorem (which is a conjecture) permits black holes to have only three observable parameters, one of which is electric charge. In the absence of a dense atmosphere, the amount of charge needed to induce dielectric breakdown in the local neighborhood of a black hole is expected to be very large. Therefore, it is reasonable to suppose that "cataclysmic" and "one-off" FRB events are black hole lightning. Dielectric breakdown of accretion matter is one possible mechanism. Dielectric breakdown of the quantum vacuum is an exotic mechanism which might be investigated.

As a work in phenomenology, one would assemble known radio models of terrestrial lightning and then compute the characteristics of black hole lightning needed to produce the observed radio fluxes at cosmological distances. A few known repeating FRB sources may be understood as black hole lightning storms. Planetary storm clouds are known not to totally discharge in single lightning bolts, so some constraint mechanism would be introduced to explain the possible incomplete electric discharge of a black hole upon a single FRB event.

Appendix A: The Origin of $\hat{M}^3$

The original motivation for $\hat{M}^3$ was only a requirement for some third order operator needed to generate the $(\Phi\pi)^3$ term appearing in $\alpha^{-1}_{\text{MCM}} = (\Phi\pi)^3 + 2\pi$. However, the third order operator became independently useful, as in Section 1 and elsewhere. For breadth, this appendix will fully review the original development of $\hat{M}^3$ in which it was conceived only as a way to force a cubed term into a theory where cubed terms usually do not appear. The first statement of $\hat{M}^3$ appeared in [30]. This was published before the construction of the unit cell [7] whose structure provides the current best framework for understanding $\hat{M}^3$.

What follows is the first statement of requirements for $\hat{M}^3$ as given in [30]. For consistency with present notations, the original symbol $\aleph$ from [30] is replaced with $\mathcal{A}$. Following the excerpt, we will carefully review what was written.

> "If the observer's proper time is t_0[,] we can write the following with certainty.
>
> $$\begin{aligned} \text{Past} &:= [t_{\text{min}}, t_0) \\ \text{Present} &:= [t_0] \\ \text{Future} &:= (t_0, t_{\text{max}}] \quad . \end{aligned} \tag{A.1}$$
>
> "In General Relativity there is no inertial frame but one is assumed and L^2 is the vector space of this approximation. Unitary evolution [*of charged particles*] in this space is characterized by orders of [*the fine structure constant*]. This number should be a direct prediction of a complete Quantum Theory. A finely structured theory is needed, one which does not reside in the Hilbert space $\mathcal{H}$ alone. To be precise, define a Gelfand triple $\{\mathcal{A}, \mathcal{H}, \Omega\}$ where each set contains a Minkowski picture $\mathbf{S}$.
>
> $$\begin{aligned} \mathcal{A} &= \left\{ x^\mu_- \in \mathbf{S} \middle| t_{\text{min}} < t < t_0 \right\} \\ \mathcal{H} &= \left\{ x^\mu \in \mathbf{S} \middle| t = t_0 \right\} \\ \Omega &= \left\{ x^\mu_+ \in \mathbf{S} \middle| t_0 < t \leq t_{\text{max}} \right\} \quad . \end{aligned} \tag{A.2}$$
>
> "The Minkowski diagram gives a clear illustration. The past and future light cones define the half spaces $\mathcal{A}$ and Ω[,] and the hypersurface of the present is a delta function $\delta(t - t_0)$. The present is defined according to the observer so it is an axiom of this interpretation that the observer is

isomorphic to the δ function. Our task is how to reconcile calculations in $\mathcal{H}$ with the actual dynamics of Nature proceeding around us and through us in $\aleph$ and Ω. To this end[,] define an operator $\hat{M}^3$ that is non-unitary and complimentary to the unitary evolution operator $\hat{\mathcal{U}}$.

$$\begin{array}{lcl} \hat{\mathcal{U}} : \mathcal{H} \mapsto \mathcal{H} & \text{and} & \hat{M}^3 : \mathcal{H} \mapsto \Omega \mapsto \mathcal{A} \mapsto \mathcal{H} \\ \hat{\mathcal{U}} := \partial_x \quad , & & \hat{M}^3 := \partial_t \quad ." \end{array} \tag{A.3}$$

The above appeared in [30] only as a segue into the main result of the short paper titled "Derivation of the Fine Structure Constant." While terse brevity is a hallmark of an academic writing style, the brevity of the segue has been cited as rendering the entire work nonsensical. The purpose of this appendix is in part to refute such claims. To that end, the reader is encouraged to understand that these few words excerpted from the beginning of [30] were written only to introduce the main result that $(\Phi\pi)^3 + 2\pi \approx 137$ very nearly replicates the accepted value for α^{-1}_{QED}. This result does not hinge on any of the material quoted above yet detractors cite the abrupt progression through the introduction as if it nullifies the main result of the paper whose title is as stated: derivation of the fine structure constant.

The second sentence of the excerpt about the assumption of inertial frames means that although flat space does not exist, it is assumed to exist. The L^2 space of position space wavefunctions is usually assumed on a flat spacetime background. One does quantum mechanics in an implicit Lorentz frame even if general relativity is not considered. In QED, a relativistic extension of quantum mechanics, α characterizes the interaction between photons and charged particles. Various generating functionals of amplitudes for processes involving charged particles may be decomposed as power series in α. The unitary evolution of such particles is foremost among those things which are described with quantum theory, as in the excerpt's third sentence. The main purpose of these remarks is not to derive the FSC. Instead, they are a segue into what would otherwise be a single line reporting the paper's main result: $(\Phi\pi)^3 + 2\pi \approx 137$ may be of interest to those who wonder about where α comes from.

Dirac said finding the origin of this number is, "the most important unsolved problem in physics," and Feynman wrote the following [86].

> "It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good

> theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: ***is it related to pi*** [*emphasis added*] or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the 'hand of God' wrote that number, and 'we don't know how He pushed his pencil.' We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!"

While all good physicists worry about this number whose origin *may be related to* π, many readers of [30] expressed no interest in it, preferring instead to fixate on a few of the tangential introductory remarks. The main purpose of [30] was to demonstrate that a certain "dance" with a third order operator does the trick quite nicely. At the end of the paper, it was suggested that if this dance is real and not wrongly contrived, then there should exist observables correlated with delay. The main point of the paper, however, was to report that there exists a previously unconsidered type of dance which outputs the requisite number to within an accuracy that can probably be reconciled via theoretical restructuring, as in Section 1.9.

As is usual in physics, a segue giving some context was given at the beginning of the paper. As is not usual in physics, detractors cite the segue as if it was something other than a few brief words given for context.[1] Furthermore, the delay experiment which was suggested to support the context returned an affirmative confirmation [32]. If the experimental prediction had not been confirmed, the paper's main result would have stood on its own: $(\Phi\pi)^3{+}2\pi$ might be of interest to those interested in the "most important unsolved problem in physics."

The next item in the excerpt regards the introduction of rigged Hilbert space. The reasons for doing quantum theory in rigged Hilbert space are well known. For instance, de la Madrid writes the following [59].

> "Nowadays, there is a growing consensus that the [*rigged Hilbert space*], rather than the Hilbert space alone, is the natural mathematical setting of Quantum Mechanics."

[1] A certain formula for the Riemann ζ function was given at the beginning of [48] wherein the architecture of the neighborhood of infinity and an eventual formal negation of the Riemann hypothesis were developed. Many readers of [48] fixated on the lack of a caveat clarifying that the formula was not defined everywhere on the complex plane while completely ignoring the paper's main result. Then, as in [30], the limitation on the domain of validity for that formula had absolutely no impact on the paper's topic or main results. It is not required to reinvent the wheel each time a scientific paper is written, and the formula was presented only to sketch a setting for the main result.

Nothing new about rigged Hilbert space is written in [30]. Position eigenstates don't exist in Hilbert space, but definite location at a point in Minkowski space **S** can only be represented in quantum theory with a position eigenstate. Since one uses such eigenstates (and similar) very often in the course of doing quantum mechanics, one would adopt a state space which does not preclude their existence.

One valid criticism of [30] regards a notational deficiency. We fail to distinguish with separate labels the state spaces in the RHS from the subsets of **S** identified with the past, present, and future. This deficiency has been remedied in subsequent work with the addition of a tick mark to distinguish $\{\mathcal{A}',\mathcal{H}',\Omega'\}$ from the manifolds $\mathcal{A}$, $\mathcal{H}$, and Ω. In [30], it is clear that $\{\mathcal{A},\mathcal{H},\Omega\}$ is a rigged Hilbert space, and it is clear that the given $\mathcal{A}$, $\mathcal{H}$, and Ω are subsets of Minkowski space. However, the non-ticked notation relies on the reader's ability to differentiate between state spaces and geometric manifolds. The statement that each set (a state space is a set of vectors equipped with an inner product) contains a Minkowski picture is imprecise. The state spaces are said to contain the coordinate spaces because the coordinate spaces are the domains of the functions which represent the states. It might have been stated more clearly that if ψ is a function of $x^i_- \in \mathcal{A}$, then $\psi \in \mathcal{A}'$. Still, the paper's main result was that a third order operator can output the value $(\Phi\pi)^3$ required for α_{MCM}, and that observed delay correlations would lend further support to the way $\hat{M}^3$ was hypothesized. The distinction of the domain of each function space was not very important for the main result, and the detail was glossed over.

Moving on to the next item in the excerpt, the hypersurface of the present is *given by* $\delta(t-t_0)$, as per usual. Some readers of [30] insist that they cannot, could not, or would not understand the obvious relationship between the Dirac δ function and a surface selected from a bulk. It is claimed that the absence of further words such as "given by" overwhelmed and destroyed their knowledge of the only possible relationship between a δ function and a surface. Regarding the usual ability of a scientific reader to infer, consider the definition of the Dirac δ function published by Proceedings of the Royal Society of London in 1927 [231].

> "One cannot go far in the development of the theory of matrices with continuous ranges of rows and columns without a notation for that function of a c-number x that is equal to zero except when x is very small, and whose integral through a range that contains the point $x=0$ is equal to unity. We shall use the symbol $\delta(x)$ to denote this function, i.e.: $\delta(x)$ is defined by
>
> $$\delta(x) = 0 \quad , \quad \text{with} \qquad x \neq 0 \quad , \tag{A.4}$$

and
$$\int_{-\infty}^{\infty} x\,\delta(x) = 1 \quad .\text{''} \tag{A.5}$$

None of Dirac's readers reached this definition and stopped with a declaration, "This is nonsense! Matrices have discrete rows by construction, so there is no such thing as a matrix with continuous rows." None stopped reading and put Dirac's paper into the trash declaring, "The integral symbol only has meaning when it appears with the differential of an integration variable such as dx. Clearly this fool Dirac, who does not understand even the most basic principles of calculus, is wasting pages in the journal where I have found his paper!" It is suggested that the main difference by which Dirac's readers were able to infer a dx in the integral over $\delta(x)$ while others were not able to infer the relationship between a δ function and a surface is that Dirac's readers were reading with an intention to understand while others were reading about the MCM with an intention to say that the MCM is worthless, and that the author is a poor pretender or worse.

The hypersurface of the present is given by a δ function in the way that one might select the volume V of all of space from the volume VT of all of spacetime. One inserts $\delta(t-t_0)$ into $\int d^4x$. The selection of such surfaces by δ functions is standard, and this is what was meant in [30] when it was said that the hypersurface of the present *is* a δ function, rather than that it *is given by* one. What we usually do with the hypersurface of the present in QM is to integrate over all of it. Quantum mechanics usually ignores d^4x, but the dance prescribed in [30] is such that we need to differentiate among the d^3x at various x^0. The intended readership was assumed to have some familiarity with the physicist's basic mathematical toolbox, but many detractors have admitted no such familiarity. While it is true that the surface is not the δ function itself identically, the reader is given a choice by the brevity. They may understand the relationship between surfaces and δ's, or they may choose not to. Furthermore, the hypersurface of the present being given by a δ function has almost nothing to do with the paper's main result. It is only mentioned to compare the present moment's quality of singular thinness to the extended bulk of the past and future, and to complement, thereby, the stated division of

$$\text{Past} := [t_{\min}, t_0) \quad , \quad \text{Present} := [t_0] \quad , \quad \text{and} \qquad \text{Future} := (t_0, t_{\max}] \quad . \tag{A.6}$$

The observer is said to be *isomorphic* to the δ because the δ that selects the hypersurface of the present is comoving in spacetime with the observer. If the observer's proper time is t_{now}, that shows up in the stated mechanism as $\delta(t-t_{\text{now}})$. Isomorphic means "corresponding or similar in form and relations," and the association of the observer at proper time t_{now} with $\delta(t-t_{\text{now}})$ is exactly that.

Now we have worked through the *introductory remarks* in [30]. The remainder of the excerpt proposes the $\hat{M}^3$ operator whose non-unitarity and functioning are discussed in Section 1 of the present paper. After briefly explaining the relationships among $\{\mathcal{A},\mathcal{H},\Omega\}$, the mathematical property of $\hat{M}^3$ was stated as

$$\hat{M}^3:\mathcal{H}_1'\to\Omega'\to\mathcal{A}'\to\mathcal{H}_2' \quad . \tag{A.7}$$

It was suggested that the mechanism proposed for $\hat{M}^3$ would result in observable delay correlations, and these were observed in the BaBar data forthwith [32]. The fixation of detractors on the terseness of [30] belies a low comprehension if not a malicious intent to wrongfully naysay. A positive reader should have come away with the understanding that α^{-1}_{MCM} can be extracted from some rather ordinary quantum mechanical formalism, that α^{-1}_{MCM} and α^{-1}_{QED} differ by about 0.4%, and that observed delay correlations were expected to serve as experimental support. In the remainder of this appendix, we will continue a critical review of [30]. The intention is to address all possible criticisms that a non-positive or overly pedantic reader might seize upon in lieu of the main results.

To contextualize the FSC result itself, first consider that the MCM is such that the universe is like a quantum particle. Since the universe contains smaller quantum particles of its own, an apparent scale invariance and self-similarity in the model directed this writer's attention toward fractal models of cosmology. Coming quickly to the prolific body of work due to El Naschie, a formula was encountered for the fractal dimension of a Cantorian spacetime [220]:

$$D=4+\varphi^3 \quad , \quad \text{where} \qquad \varphi=\frac{|1-\sqrt{5}|}{2} \quad . \tag{A.8}$$

This formula profoundly attracted this writer's attention, as described in [95]. The formula

$$\alpha^{-1}_{\text{MCM}}=2\pi+\big(\Phi\pi\big)^3\approx 137 \quad , \tag{A.9}$$

was quickly obtained by the ansatz method. The example of the 2D box given in [30] was devised to support an explanation for where such a number might come from. The sides of the box—a duration D and a length L—were chosen to satisfy $\Phi D=2L$. Written as $D=2\varphi L$, this is in the same general form of $C=2\pi R$ giving the circumference of a circle in terms of the radius.[1] This is somewhat interesting as

[1] In Section 1.2.4, we assumed that $\mathcal{H}$ was spanned by one unit of time so that we could use the locations of $\mathcal{A}$ and Ω at $\chi^4_-=-\varphi$ and $\chi^4_+=\Phi$ to identify in the unit cell a pair of $\Phi\times 1$ and $1\times\varphi$ golden rectangles. However, x^0 rightfully spans infinitely far into the past and infinitely far into future, so we would assume that the length of the $\mathcal{H}$-brane in

a geometric confluence, and it was stated as such in [30]. However, it went unstated that (A.9) was conceived as a *circularization* of the *rectangular* (A.8), meaning $4 \to 2\pi$ and $\phi^3 \to (\Phi\pi)^3$. In that way, in the context of the author's thinking, the confluence of the dimensions of the 2D box mimicking $C = 2\pi R$ was slightly more significant than what was recorded in [30].

The inclusion of t for one of the sides of the box was a hard concept because t is only used for time evolution in quantum mechanics. States in Hilbert space do not depend on t as they do the x associated with the L side of the box. However, the subsequent introduction of the χ^4 variables as a second form of time sidesteps the problem of double usage for t. The 2D box was eventually replaced by the unit cell, and the box spanned by space and time later became important for the MCM particle scheme, as in Section 0.3 and [6]. The general idea for an operator on a state in a box which should return α_{MCM}^{-1} also remains.

Continuing with a review of the material in [30], the well known wavefunction of a particle in a 2D box with sides D and L is

$$\psi_{nm}(x,t) = \frac{2}{\sqrt{DL}} \sin\!\left(\frac{n\pi x}{L}\right) \sin\!\left(\frac{m\pi t}{D}\right) \quad . \tag{A.10}$$

A box spanned by one dimension of space and one dimension of time is a 2D universe, so putting the particle in this box was like putting it in a finite model of the universe. Likewise, one of the main purposes of the modern unit cell is to put a universe, possibly even a universe extending infinitely far in its physical coordinates, inside an abstract box of finite dimension. Having fixed the box' aspect ratio $D = 2\varphi L$, the duration was chosen as φ. The chosen dimensions are such that

$$\psi_{nm}(x,t) = 2\sqrt{2\Phi}\,\sin\!\big(2n\pi x\big)\,\sin\!\big(\Phi m\pi t\big) \quad . \tag{A.11}$$

This is not a simultaneous eigenvector of ∂_x and ∂_t^3 as would be required for equation (19) in [30]:

$$\hat{\Upsilon}\psi_{11} = \big(\partial_x + \partial_t^3\big)\psi_{11} = \alpha_{\text{MCM}}^{-1}\,\psi_{11} \quad , \tag{A.12}$$

where $\hat{\Upsilon} = \partial_x + \partial_t^3$ reflects $\hat{\mathcal{U}} := \partial_x$ and $\hat{M} := \partial_t$. Instead, these partial derivatives operate on (A.11) as

$$\partial_x\psi_{11}(x,t) = 2\pi\phi_1(x,t) \quad , \quad \text{and} \qquad \partial_t^3\psi_{11}(x,t) = \big(\Phi\pi\big)^3\phi_2(x,t) \quad , \tag{A.13}$$

where ϕ_1 and ϕ_2 demonstrate that $\psi_1 1$ is incompatible with the eigenvalue equation

the x^0 direction is $2\widehat{\infty}$. Assuming that the chronological and chirological coordinates are on different levels of aleph scaled by $\widehat{\infty}$, $D = 2\widehat{\infty}$ in $\mathcal{H}$ and $L = \Phi\widehat{\infty}$ in Σ^+ satisfy the ratio $D = 2\varphi L$.

in (A.12). The exponential, not the sine, is the eigenfunction of the derivative. Since α^{-1} is observable, quantum theory suggests that it should be an eigenvalue of an operator's eigenvector, but this detail was neglected in [30].

The equation $\hat{\Upsilon}=\hat{\mathcal{U}}+\hat{M}^3$ was written in [30], but we have used it as $\hat{\Upsilon}=\partial_x+\partial_t^3$. This could have been better clarified. The statement $\hat{\mathcal{U}}:=\partial_x$ refers to

$$\hat{\mathcal{U}}(t,t_0)=\exp\left\{\frac{-i\hat{H}\big(t-t_0\big)}{\hbar}\right\}=\exp\left\{\frac{-i\big(t-t_0\big)}{\hbar}\left[\frac{\hat{p}^2}{2m}+\hat{V}(\hat{x})\right]\right\}\quad ,^1 \tag{A.14}$$

wherein the $\hat{H}$ depends on $\hat{p}\propto\partial_x$. To get 2π out of ψ as needed for $2\pi+(\Phi\pi)^3$, we have operated with ∂_x. Since $\hat{\mathcal{U}}$ uses $e^{\partial_x^2}$, there would be an implicit square root somewhere, and a logarithm would be required to use $\hat{\mathcal{U}}$ for returning 2π. For instance, equations (7) and (8) in [30] were

$$\begin{aligned}\hat{\mathcal{U}}\big|\psi\big\rangle &:= \partial_x\big(2n\pi x\big)=2n\pi \\ \hat{M}\big|\psi\big\rangle &:= \partial_t\big(\Phi m\pi t\big)=\Phi m\pi \quad ,\end{aligned} \tag{A.15}$$

but the time evolution operator $\hat{\mathcal{U}}$ (given a time-independent Hamiltonian and an energy eigenstate ψ_E) is such that

$$\hat{\mathcal{U}}\big|\psi_E\big\rangle=e^{-iEt/\hbar}\big|\psi_E\big\rangle \quad . \tag{A.16}$$

The := symbol has been used in [30] to suppress the fact that the value for α^{-1}_{MCM} would probably appear as an exponent. As [30] is written, all such details pertaining to an exact arithmetic are relegated to the catch-all := symbol. However, the hard functioning of the main result is given by the the strict equality

$$\alpha^{-1}_{\text{MCM}}=2\pi+\big(\Phi\pi\big)^3 \quad . \tag{A.17}$$

The context of the 2D box was reverse engineered to fit this result. Similarly, Schrödinger's initial publication of his equation [80] gave its context as being derived from the stationary action principle. That reasoning for coming to the Schrödinger equation has not stood the test of time even while the equation itself has survived. Likewise, the context of the 2D box proffered in [30] is no longer an attractive path toward arriving at the equation for α^{-1}_{MCM}. Still, the box is important for this appendix.

The operator $\hat{\Upsilon}$ in (A.12) requires a simultaneous eigenfunction of ∂_x and ∂_t^3. One

[1] This form of $\hat{\mathcal{U}}$ is developed in Appendix B.

such function is

$$\Psi_{nm}(x,t) = Ae^{i\pi(2nx+\Phi mt)} \quad . \tag{A.18}$$

Ψ_{nm} would be obtained by rescaling x and t to be small relative to the dimensions of the box. Far from the edges of the universe-as-a-box, the solutions are plane waves in an ordinary way of approximating physics.[1] In this case, one might have simply supposed plane waves with the given wavenumber and frequency without invoking the context of a box at all. It is true that free particle plane wave solutions are essentially the opposite of particle-in-a-box solutions (Section 1.7.3), but it also true that plane waves in the universe are ultimately constrained to be particle in a box states due to the L^2 condition of square integrability.[2] In any case, observable operators have real-valued eigenvalues, and (A.18) does the trick with operators $-i\partial_x$ and $-i\partial_t^3$:

$$\hat{\Upsilon} = -i\big(\partial_x + \partial_t^3\big) \qquad \Longrightarrow \qquad \hat{\Upsilon}\Psi_{11} = \Big[2\pi + \big(\Phi\pi\big)^3\Big]\,\Psi_{11} \quad . \tag{A.19}$$

Overall, the heavy reliance on the := symbol in [30] was purposed to avoid tangential details. The unstated point in presenting the 2D box was that it was reasonable to expect that a sufficient eigenstate should exist due to the low algebraic complexity of the formula for α^{-1}_{MCM}. Noting that even the inclusion of this present paragraph would have increased the word count of [30] by 10% or so, certain details were omitted. To facilitate the structure of the paper, a *toy model* was constructed wherein one obtains α^{-1}_{MCM} as the eigenvalue of an operator. In hindsight, it may have been better to directly suppose the plane waves in (A.18) than to try to extract the geometric setting of a box. On the other hand, the initial ideation for a box led to the box-like structure of the MCM unit cell which has been useful for continued inquiry.

Regarding the excerpt below, non-standard language around (A.20), (A.21), and (A.22) also deserves attention.

> "We have defined a unitary time evolution operator and a non-unitary one. Assume the correct evolution operator is the sum of a unitary part

[1] Confinement to a finite volume induces quantization on a state's wavenumber, but for applications in which the wavelength is much smaller than the dimension of the box, which is also called being *far from the edges*, one often ignores the quantization to suppose that the state has a continuous wavenumber and extends infinitely far. This approximation is generally valid when the range of wavenumbers considered is such that the wavelength is everywhere small compared to the confinement dimension, *and* when the region under consideration is far from the edges of the box. For visualization purposes, one understands that increasing the quantum number on a particle-in-a-box state adds maxima to the sinusoidal wavefunction. When the number of maxima is very large, it is often safe to assume that the quantum number becomes continuous. It is also safe to ignore the non-plane wave boundary condition that the states must vanish at the edges of the box if one is considering a region where the wavefunction is separated from the boundary by a large number of local maxima. Thus, one assumes plane waves in an ordinary way of approximating physics.

[2] This case for assuming plane waves follows the conditions in the previous footnote. The universe is large compared to a typical de Broglie wavelength.

and a non-unitary part so that $\hat{\Upsilon} = \hat{\mathcal{U}} + \hat{M}^3$

$$\big\langle\psi;x,t\big|\hat{\Upsilon}\big|\psi;x,t\big\rangle = \big\langle\psi;x,t\big|\hat{\mathcal{U}}\big|\psi;x,t\big\rangle + \big\langle\psi;x,t\big|\hat{M}^3\big|\psi;x,t\big\rangle \tag{A.20}$$

$$\big\langle\psi\big|\hat{M}^3\big|\psi\big\rangle := \int \psi^*(t)\,\delta(t)\,\partial_t^3\psi(t)\,dt \quad , \tag{A.21}$$

$[w]$here the inclusion of $\delta(t)$ fixes the observer at $[t=0]$. The integral over all times will trace a path through $\mathcal{A}$, $\mathcal{H}$, and Ω. To use the integrand $f(t)\delta(t)[,]$ we must employ the familiar method from complex analysis.

$$\int_{-\infty}^{\infty}\delta(t)\,f(t)\,dt = \int_0^{t_{\max}} f(r,0)\,dt + \int_0^{\pi} f(\infty,\phi)\,d\phi + \int_{t_{\min}}^{0} f(r,\pi)\,dt \quad . \tag{A.22}$$

This method is an outstanding logical proxy for the process [*Present*$\to$ *Future*$\to$*Past*$\to$*Present*]."

The usual definition for an expectation value is

$$\big\langle\hat{Q}\big\rangle = \int d^3x\,\psi^*(\mathbf{x})\,\hat{Q}\,\psi(\mathbf{x}) \quad , \tag{A.23}$$

but the excerpt contains

$$\big\langle\hat{M}^3\big\rangle := \int dt\,\psi^*(t)\delta(t)\partial_t^3\psi(t) \quad . \tag{A.24}$$

Keeping in mind that t has been replaced with χ^4 in subsequent work, the purpose of this non-standard definition of the expectation value was to induce the piecewise structure on t which is now found in $\{\chi^4_+, \chi^4_-, \chi^4_\varnothing\}$. The $:=$ symbol is used in (A.24) to highlight only the new MCM part of the integral while ignoring the spatial part that should take its usual form, as in (A.23). If the $\delta(t)$ appearing in (A.24) is the Dirac δ function, we obtain

$$\big\langle M^3\big\rangle := \big(\Phi\pi\big)^3\int dt\,\psi^*_{11}(t)\delta(t)\psi_{11}(t) = \big(\Phi\pi\big)^3\big|\psi(0)\big|^2 = \big(\Phi\pi\big)^3 \quad , \tag{A.25}$$

as expected. There is no need for the separated path of integration in (A.22).

At the time of the publication of [30] in 2011, this writer was under the wrong impression that there exists a class of spike functions called δ functions, one which is named after Dirac. A primitive definition of the Dirac δ function is

$$\delta(x-x_0)=\begin{cases}\infty & \text{for } x=x_0 \\ 0 & \text{otherwise}\end{cases} \quad , \tag{A.26}$$

but the $\delta(t)$ appearing in (A.24) was purposed as a generalized spike function

$$\bar{\delta}(x-x_0)=\begin{cases}\infty & \text{for } x=x_0 \\ 1 & \text{otherwise}\end{cases} \quad . \tag{A.27}$$

It is entirely reasonable that a reader would assume that $\delta(t)$ is the Dirac δ function from which the use case for a vanishing path of integration at infinity as in (A.22) does not follow in an intuitive way. Now we will describe how the path of integration in (A.22) follows from (A.27).

The expectation value for $\hat{M}^3$ requires an integral over all of time, but we have inserted a pole at $t=0$ with $\bar{\delta}(0)$. Rather than to revise the definition of the expectation value, the pole is meant to separate time into the three regimes of Past, Present, and Future defined at the outset of [30]. A common method in physics for dealing with a pole along the path of integration in integral $\mathcal{I}$ is to move the pole off of the path by the addition of an imaginary infinitesimal somewhere. Then one forms a closed path integral with $\mathcal{I}$ and another path of integration at infinity along which the L^2 condition makes the integrand vanish at every point. Cauchy's residue theorem is applied to solve for $\mathcal{I}+0$ as $2\pi i$ times the residue at the pole.

The meaning of (A.22) is that the closed path of integration in the Cauchy formula may be parsed as the process for $\hat{M}^3$ when we integrate:

- from $t=0$ to $t_{\text{max}}=\infty$ as Present$\to$Future,
- from t_{max} along a path at infinity to $t_{\text{min}}=-\infty$ as Future$\to$Past,
- and finally from t_{min} back to $t=0$ as Past$\to$Present.

Although the unit cell was not constructed when [30] was published, these three paths along the closed Cauchy curve are like $\mathcal{H}\to\Omega$, $\Omega\to\mathcal{A}$, and $\mathcal{A}\to\mathcal{H}$. The current parameterization of this path in terms of χ^4 between two $\mathcal{H}$-branes may yet be simplified with Cauchy's theorem and a pole located in or near $\mathcal{H}$ or $\varnothing$. A winding number is easily added to the Cauchy curve to identify the integration's start and endpoints with two different instances of $\mathcal{H}$ rather than with each other.

Aside from presenting the main result, the remainder of [30] develops the algebra from which Einstein's equation would be derived almost a year later in [3]. It was emphasized heavily in subsequent work that the MCM derivation of Einstein's equation may appear to have been goal-sought, or reverse engineered, but no such thing was the case. The algebra in which Einstein's equation appeared was assembled long before the GR result was found and reported in [3]. The comment on Palev statistics in [3] reflects a comment made to this writer by Finkelstein (Section 33) who had gone into professor emeritus status shortly before this writer was accepted as a PhD candidate and awarded a prestigious fellowship at Georgia Tech. The possible relevance of Palev statistics was never investigated and may have been a monkey wrench thrown into the works by a notorious and miserly detractor of the MCM.

As one further remark on the algebra developed in [30], the reader is invited to notice that [30]'s equation (18) is a rich algebraic structure indeed. It is not cited as a thesis in the main body of this paper, but this algebra should be reconstructed in the language of Galois theory, if possible. The structure is quite rich. This writer has never seen another like it, but that may reflect this writer's limited exposure to abstract algebra.

The conclusion of [30] gives a suggestion to look for delay correlations in particle collider data.

> "If variations in α can be detected by varying the delay between an event and its measurement in an experimental apparatus that will strongly support the ideas presented here."

Very soon after this prediction was published, the BaBar collaboration discovered time reversal symmetry violation through delay correlations in their previously collected data [32]. In earlier work on the MCM [31], dark energy had been described as a delay correlation of sorts, and the algebraic structure around [30]'s equation (15)—the $\varphi^{**}\neq\varphi$ property of $\mathbb{C}^*_\pm$—was meant to break time reversal symmetry. Particularly, if the space of states in the past is different than the space of states in the present, it was suspected that the duration between an event and its observation would have observable correlations. Since the experimental quantity in context was the FSC, it was suggested that the delay correlations would manifest in its observed value.

The following is a summary of the prominent issues with the original formulation in [30].

- The well known state ψ of a quantum particle in a 2D infinite square well was employed as
$$\hat{\Upsilon}\big|\psi\big\rangle = \big(\partial_x + \partial_t^3\big)\big|\psi\big\rangle = \alpha^{-1}_{\text{MCM}}\big|\psi'\big\rangle \quad . \tag{A.28}$$

The tick mark showing that ψ is not an eigenstate of $\hat{\Upsilon}$ was omitted. The state

$$\psi_{nm} = \frac{2}{\sqrt{LD}} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi t}{D}\right) \quad . \tag{A.29}$$

of a 2D particle in a box spanned by x and t in respective dimensions L and D (length and duration) is not an eigenstate of $\partial_x + \partial_t^3$. Since α is observable, the general axiomatic framework of QM suggests that a Hermitian operator should return α^{-1} when acting on an eigenvector. The given ψ_{nm} fails to satisfy an eigenvalue equation with $\hat{\Upsilon}$ as defined.

- α^{-1}_{MCM} is returned only upon choosing $L{=}1/2$ and $D{=}2\varphi L$. The fixed dimensions of the box are associated easily enough with the fixed abstract dimensions of the unit cell, but no explanation for this ratio was proposed.
- α^{-1}_{MCM} is returned as the $nm{=}11$ eigenvalue of ψ_{nm}, but there is no ready interpretation for the other nm eigenvalues. α^{-1} should be returned by an *ontological*, or unique, eigenstate without an unbounded spectrum of other values for n and m.
- While the single spatial derivative in $\hat{\alpha} = \partial_x + \partial_t^3$ was natural to the 2D box model, the full theory would have three spatial derivatives inherent to the ∇ operator. With α being rooted historically in 3D atomic physics, the initial context for one spatial dimension must be generalized to the full theory. It seems likely that this generalization would unfixably alter the 2π part of α^{-1}_{MCM}.
- In the statement $\hat{\Upsilon}{=}\hat{\mathcal{U}} + \hat{M}^3$, the relationship between $\hat{\mathcal{U}}$ and ∂_x inherent to

$$\hat{\mathcal{U}}(t, t_0) = \exp\left\{ \frac{-i\hat{H}\big(t - t_0\big)}{\hbar} \right\} \quad , \tag{A.30}$$

reflects only the simplest case of a time-independent Hamiltonian. Already, a function for extracting one linear ∂_x from $\hat{\mathcal{U}}$ seems too complicated. The generalization to a time-dependent Hamiltonian would be an analytical mess. Backing ∂_x out of the Dyson series representation of a Hamiltonian such that $[\hat{H}(t_0), \hat{H}(t_1)] {\neq} 0$ may not be possible.

Appendix B: Focused Review of Quantum Mechanics

To build up the usual operator formalism which shall be extended with $\hat{M}^3$, we will begin in the basis of position eigenstates. This is the usual path of development for QM because it connects so well with the picture of classical physics. Much of this appendix follows Sakurai and Napolitano [83].

B.1 The Translation Operator

By definition, position eigenstates are eigenvectors of the position operator

$$\hat{x}\big|x\big\rangle = x\big|x\big\rangle \quad . \tag{B.1}$$

The position operator $\hat{x}$ has a complete continuous spectrum. The completeness relation is

$$\mathbb{1} = \int dx\,\big|x\big\rangle\big\langle x\big| \quad . \tag{B.2}$$

To move a particle from position x_1 to position x_2, the machinery of quantum mechanics requires that we operate on $|x_1\rangle$ with an operator such that $|x_2\rangle$ is the result. We will call that operator the translation operator and label it $\hat{\mathcal{J}}$. Evidently, it satisfies

$$\hat{\mathcal{J}}(\Delta x)\big|x\big\rangle = c\big|x+\Delta x\big\rangle \quad . \tag{B.3}$$

This equation comes directly from the physics of motion: $\hat{\mathcal{J}}$ moves $|x\rangle$ to $|x+\Delta x\rangle$. Now it remains to reverse engineer the analytical form of the operator. Similarly, we have proposed that $\hat{M}^3$ should move $|\psi\rangle$ like so, like so, and like so, and then left determining the actual machinery of $\hat{M}^3$ to a later endeavor. This is what is done with $\hat{\mathcal{J}}$ and other operators. One conceives of an operation, labels the operator that does it, and then works out what it has to be. It is no hoax that we have written (B.3) without knowing what mathematical form $\hat{\mathcal{J}}$ might take and neither is the MCM reliance on $\hat{M}^3$ without first defining its analytical form. This is business as usual in quantum theory.

It is for a good reason that

$$\hat{M}^3\big|\psi;\mathcal{H}_1\big\rangle = c\,\big|\psi;\mathcal{H}_2\big\rangle \quad , \tag{B.4}$$

looks like (B.3). $\hat{\mathcal{J}}$ is the spatial translation operator, and $\hat{M}^3$ is another kind of translation-like operator between unit cells. $\hat{M}^3$ is necessarily more complicated than $\hat{\mathcal{J}}$ because it must be uniquely complemented by a time evolution to the later chronological time on the forward $\mathcal{H}$-brane. This added complexity is part of why $\hat{M}^3$ is posed as three separate operations, and $\hat{\mathcal{J}}$ is not.

To work out the mathematical representation of $\hat{\mathcal{J}}$ based on the physics assigned to it, first we will consider infinitesimal 1D translations:

$$\hat{\mathcal{J}}(dx)\big|x\big\rangle = c\big|x+dx\big\rangle \quad . \tag{B.5}$$

For ease in notation, we will set $c=1$. As per usual in quantum mechanics, we will explore the mathematical structure by inserting (B.2): the completeness relation. Assigning the dummy integration variable x', we have

$$\begin{aligned}\hat{\mathcal{J}}(dx)\mathbb{1}\big|x\big\rangle &= \hat{\mathcal{J}}(dx)\int dx'\,\big|x'\big\rangle\big\langle x'\big|x\big\rangle \\ &= \int dx'\,\hat{\mathcal{J}}(dx)\big|x'\big\rangle\big\langle x'\big|x\big\rangle \\ &= \int dx'\,\big|x'+dx\big\rangle\big\langle x'\big|x\big\rangle \quad .\end{aligned} \tag{B.6}$$

The quantity $\langle x'|x\rangle$ is the interpreted as the expansion coefficient of $\hat{\mathcal{J}}(dx)|x\rangle$ written in the basis of $|x'+dx\rangle$ states. That basis is merely the position basis with position measured from an origin shifted by dx, so we will introduce a coordinate transformation to shift it back. Using

$$x'' = x'+dx \qquad \Longrightarrow \qquad dx''=dx' \quad , \tag{B.7}$$

we obtain

$$\hat{\mathcal{J}}(dx)\big|x\big\rangle = \int dx''\,\big|x''\big\rangle\big\langle x''-dx\big|x\big\rangle \quad . \tag{B.8}$$

Since x' and x'' are only dummy variables, we can forget about the old x' and rename x'' as the new x':

$$\hat{\mathcal{J}}(dx)\big|x\big\rangle = \int dx'\,\big|x'\big\rangle\big\langle x'-dx\big|x\big\rangle \quad , \tag{B.9}$$

The expansion coefficient $\langle x'-dx'|x\rangle$ is called "the position space wavefunction" and written as $x(x'-dx')$. If we would have labeled the operand in (B.5) $|\psi\rangle$ rather than $|x\rangle$, then the wavefunction would be the more familiar looking $\psi(x'-dx')$. In that case, we would write

$$\hat{\mathcal{J}}(dx)\big|\psi_1\big\rangle = c\,\big|\psi_2\big\rangle \quad . \tag{B.10}$$

This should make it obvious why it can be better to label position states with their positions than Greek letters. In the ψ labeling, we would have to add some notes to say, "ψ_1 is the state of being located at x and ψ_2 is the state of being located at $x + dx$." That would be cumbersome, but it is demonstrative to emphasize that the $\langle x' - dx'|x\rangle$ appearing in (B.9) is an ordinary $\psi(x)$ despite it being written here as $x(x')$. Overall, (B.10) does not reflect the physics we have assigned to $\hat{\mathcal{J}}$ in the forthright manner of (B.5).

We have explained that inserting the completeness relation into the definition of infinitesimal translation makes the wavefunction appear, but we have not yet clarified what the wavefunction is. Since the wavefunction (in position space) is the expansion coefficient in the continuous basis (of position states), we should build up expansion in the discrete basis, and then generalize it to the continuous basis so that the wavefunction is not mysterious in any way. Then we will return to the analytical form of $\hat{\mathcal{J}}$ in the following subsection.

B.1.1 Interpretation of Basic Formalism in Quantum Mechanics

If one measures position, there is a continuum of different positions one might observe, so we say the spectrum of the position operator is continuous. Let there be an observable operator $\hat{A}$ such that there are only a finite number of quantized (discrete) values that might be observed in a measurement of observable A. In the $\{|a_k\rangle\}$ eigenbasis of $\hat{A}$, we have

$$\hat{A}\big|a_k\big\rangle = a_k\big|a_k\big\rangle \quad , \tag{B.11}$$

which mimics the eigenvalue equation for the position operator: (B.1). The difference is that there are an uncountably infinite number of positions x that one might find in a measurement of position, but there are only a finite number of a_k one might find when measuring observable A.

The fundamental idea in QM is that everything which can be observed may be represented as an operator. For a given observable, every possible value that may be found in an observation is an eigenvalue of that operator. The possible values of x touch each other, and the spectrum of $\hat{x}$ is called *continuous*, but there are numerical gaps between the a_k. Due to these gaps, the spectrum of $\hat{A}$ is said to be *discrete.* The completeness relation for discrete eigenbases is

$$\mathbb{1} = \sum_k \big|a_k\big\rangle\big\langle a_k\big| \quad . \tag{B.12}$$

If we operate on $|a_k\rangle$ with $\hat{A}$, we are guaranteed to get a_k since $|a_k\rangle$ is the eigenvector of $\hat{A}$ with eigenvalue a_k. However, sometimes one does not know ahead of time what outcome a measurement will give. To determine what will happen when we measure A on an unknown state ψ, we insert the completeness relation to *expand* ψ *in the* *eigenbasis of* $\hat{A}$ as

$$|\psi\rangle = \mathbb{1}|\psi\rangle = \sum_k |a_k\rangle\langle a_k|\psi\rangle \quad . \tag{B.13}$$

As in (B.9), we have obtained an *expansion coefficient* $\langle a_k|\psi\rangle$. This is the discrete version of (B.9)'s $\langle x' - dx'|x\rangle$, which we have called a *wavefunction*. Wavefunctions are the coefficients of expansion in a continuous basis. $\langle a_k|\psi\rangle$ is not a wavefunction, however. It is just a number. $\langle x' - dx'|x\rangle$ is a wavefunction because it contains the integration variable x'.

If a state $|\psi\rangle$ has its representation expanded in the basis of an operator with a discrete spectrum, the probability for finding the a_k eigenvalue is the absolute square of the $\langle a_k|\psi\rangle$ expansion coefficient. If $|\psi\rangle$ is expanded in a continuous basis, this cannot be the program for finding the probability of any particular eigenvalue because the total probability, 100%, cannot be divided into uncountably many small but finite probabilities. This is the reason why the continuous basis coefficients contain integration variables. By integrating over a finite interval containing an uncountably infinite number of continuous eigenstates, one obtains a finite, real-valued probability. Rather than taking the absolute square of the constant c_k, one integrates $|\psi(x)|^2$ across some interval. The aggregate 100% probability can always be divided into real-valued fractions across finitely many finite intervals. This can be tricky for the beginners who are the intended audience for this appendix. Thus, we belabor the details. It is the intention to make MCM publications so plainly accessible to beginners that even novices might see through detractors' stupid remarks and baseless criticisms. Early work in the MCM written exclusively for subject matter experts was flawed in that regard because myriad scandalmongers and blowhards could levy any criticism free from accountability to many third parties that might judge for themselves.

We have expanded ψ in the eigenbasis of the observable represented by $\hat{A}$: (B.13). Since the expansion coefficients are not functions of any variables, they must be numbers and we can simplify (B.13) as

$$|\psi\rangle = \mathbb{1}|\psi\rangle = \underbrace{\sum_k |a_k\rangle\langle a_k|}_{\text{Completeness}}\,\psi\rangle = \sum_k |a_k\rangle \underbrace{\langle a_k|\psi\rangle}_{c_k} = \sum_k c_k|a_k\rangle \quad . \tag{B.14}$$

For expansion in a continuous basis, we would have

$$|\psi\rangle = \mathbb{1}|\psi\rangle = \underbrace{\int d\alpha\,|\alpha\rangle\langle\alpha|}_{\text{Completeness}}\,\psi\rangle = \int d\alpha\,|\alpha\rangle \underbrace{\langle\alpha|\psi\rangle}_{\psi(\alpha)} = \int d\alpha\,\psi(\alpha)|\alpha\rangle \quad . \tag{B.15}$$

Having made clear the role of the wavefunction $\psi(\alpha)$ as an expansion coefficient,[1] we will continue in the example of the discrete basis. Then we will say more about the continuous basis as the context develops.

Operating on ψ with $\hat{A}$ does not represent a measurement of observable A. Rather, it yields a weighted sum of possible results of measurement. In the discrete case, the weight is the probability amplitude c_k for an eigenvalue a_k times the eigenvalue:

$$\hat{A}\big|\psi\big\rangle = \sum_k \hat{A}\big|a_k\big\rangle\big\langle a_k\big|\psi\big\rangle = \sum_k a_k\big|a_k\big\rangle c_k = \sum_k c_k a_k\big|a_k\big\rangle \quad . \tag{B.16}$$

This weighted average lends itself to the expectation value

$$\big\langle\hat{A}\big\rangle \equiv \big\langle\psi\big|\hat{A}\big|\psi\big\rangle \quad . \tag{B.17}$$

This is the average value that will be found across many measurements of A on identical ψ states. The orthonormal property of the $\{|a_k\rangle\}$ eigenbasis is such that

$$\big\langle a_j\big|a_k\big\rangle = \delta_{jk} \quad ,^2 \tag{B.18}$$

where δ_{jk} is the Kronecker δ so acting on (B.16) from the left with $\langle\psi|$ yields a pure number:

$$\big\langle\psi\big|\hat{A}\big|\psi\big\rangle = \sum_j\sum_k c_j c_k a_k\big\langle a_j\big|a_k\big\rangle = \sum_j\sum_k c_j c_k a_k \delta_{jk} = \sum_k c_k^2 a_k \quad . \tag{B.19}$$

We know a measurement of A on state ψ will yield eigenvalue a_k with probability $P_k = |c_k|^2$, so the interpretation of (B.19) is that the *expectation value* $\langle\hat{A}\rangle$ is the probability weighted average of possible outcomes when measuring A. If ψ was an eigenstate of $\hat{A}$, then all the c_k would be equal to zero for every value of k except one. Then we could write $|\psi\rangle = c_j|a_j\rangle = |a_j\rangle$ and $\langle\hat{A}\rangle = a_j$ without including the sum because there is no need to sum the terms whose coefficients are zero.

Unless ψ is an eigenstate of $\hat{A}$, operation by $\hat{A}$ does not return any one value of a_k even though a lab measurement of A *will* return one and only one a_k. This shows what it means when we say that the collapse of the wavefunction is implemented in an ad hoc way in quantum mechanics. For states expanded in discrete bases, collapse

[1] $\psi(x)$ is the wavefunction when the continuous eigenbasis α is the position eigenbasis.

[2] Orthogonal means $\langle a_j|a_k\rangle = 0$ when $j \neq k$ and normalized means $\langle a_j|a_k\rangle = 1$ when $j = k$. Orthonormal means orthogonal and normalized.

looks like

$$
\begin{aligned}
\big|\psi\big\rangle_{\text{discrete}} &= \sum_k \big|a_k\big\rangle\big\langle a_k\big|\psi\big\rangle \\
&= \sum_k c_k\big|a_k\big\rangle \qquad \xrightarrow{\text{measurement}} \qquad \big|a_j\big\rangle \quad .
\end{aligned}
\tag{B.20}
$$

One should compare this to (B.16) which shows that operating with $\hat{A}$ on $|\psi\rangle$ does not reduce the state to a single eigenstate unless the initial state was an eigenstate. Reflecting the lack of a natural mathematical operation for wavefunction collapse upon measurement, the long labeled arrow shows that collapse happens *somehow*. In practice, after obtaining eigenvalue a_k in an experiment, the observer will use a $\hat{\mathcal{P}}_k$ projection operator to update ψ:

$$
\hat{\mathcal{P}}_k\big|\psi\big\rangle = \big|a_k\big\rangle \quad . \tag{B.21}
$$

In the continuum, the same collapse behavior is written

$$
\begin{aligned}
\big|\psi\big\rangle_{\text{continuous}} &= \int dx'\,\big|x'\big\rangle\big\langle x'\big|\psi\big\rangle \\
&= \int dx'\,\psi(x')\big|x'\big\rangle \qquad \xrightarrow{\text{measurement}} \qquad \psi(x') = \delta(x'-x_0) \quad ,
\end{aligned}
\tag{B.22}
$$

where $\delta(x'-x_0)$ is the Dirac δ function representing the x_0 position eigenstate in position space. This δ function makes the QM of continuous observables somewhat (or massively) more complicated than the QM of discrete observables. In the discrete case, the expansion coefficients for a particular basis were just the numbers $c_k \in \mathbb{C}$ whose squares are postulated to return real-valued probabilities. In the other case, they are differentials that need to be integrated. Namely, there is no k such that we much ask about a finite probability for being located at x_k, so instead we ask about the probability for being found between x_k and x_j.

The discrete-continuous correspondence $c_k \leftrightarrow \psi(x)$ yields the following probability structure:

$$
P_k = \big|\big\langle a_k\big|\psi\big\rangle\big|^2 = \big|c_k\big|^2 \quad \longleftrightarrow \quad P(x_k)dx' = \big|\big\langle x_k\big|\psi\big\rangle\big|^2\,dx' = \big|\psi(x_k)\big|^2\,dx' \quad . \tag{B.23}
$$

The dx' tells us that the probability of observing state $|\psi\rangle$ with exact continuous parameter x_k is infinitesimal. In practice, it is not possible to measure ψ at mathematical point x_k due to resolution limits on physical devices, general principles of Heisenberg uncertainty, and ultimately Planck scale effects. While this writer is always eager to step forward with criticisms of quantum theory, (B.23) is a beautiful example of its robust power. The probability for observing a particle at a point is less than any positive real number. We would like to build devices that might detect particles at points, but such devices do not exist, and QM says they cannot exist. This is a great success among the shortcomings this writer is prone to highlight.

Since probability is dimensionless, the expansion coefficients $\psi(x)$ in the continuous basis have to have units of $[\text{meters}]^{-1/2}$ to cancel the units of dx'. These units are reflected in the normalization conditions

$$\big\langle\psi\big|\psi\big\rangle = \big\langle\psi\big|\mathbb{1}\big|\psi\big\rangle = \sum_k \big\langle\psi\big|a_k\big\rangle\big\langle a_k\big|\psi\big\rangle = \sum_k c_k^* c_k = \sum_k \big|c_k\big|^2 = 1 \tag{B.24}$$
$$\big\langle\psi\big|\psi\big\rangle = \big\langle\psi\big|\mathbb{1}\big|\psi\big\rangle = \int_{-\infty}^{\infty} dx' \,\big\langle\psi\big|x'\big\rangle\big\langle x'\big|\psi\big\rangle = \int_{-\infty}^{\infty} dx' \,\big|\psi(x')\big|^2 = 1 \quad .$$

It follows from the units that $|\psi(x)|^2$ cannot be a real probability like $|c_k|^2$. Probability is dimensionless, but $|\psi(x)|^2$ is dimensionful. Calling attention to this radical alteration of the structure for the eigenbases of operators with continuous spectra, the expansion coefficient $\big\langle x\big|\psi\big\rangle = \psi(x)$ is called the position space wavefunction rather than simply an expansion coefficient. We often call the position space wavefunction "the wavefunction." The important thing to know about wavefunctions is that they are the infinite number of expansion coefficients needed to expand an abstract state ket $|\psi\rangle$ in the infinite eigenbasis of some observable with a continuous spectrum. For $x_k \in (x_1, x_2)$, $\psi(x_k)$, which is the function $\psi(x)$ evaluated at x_k, is the expansion coefficient of the $|x_k\rangle$ basis vector in the representation of $|\psi\rangle$ in the eigenstates of the $\hat{x}$ operator.

For each of an uncountably infinite number of unique x in the spectrum of $\hat{x}$, there is a corresponding expansion coefficient $\psi(x)$. Mirroring the discrete expansion

$$\big|\psi\big\rangle_{\text{discrete}} = \sum_k c_k\big|a_k\big\rangle = c_1\big|a_1\big\rangle + c_2\big|a_2\big\rangle + c_3\big|a_3\big\rangle + \ldots \quad , \tag{B.25}$$

we would like to write the continuous expansion as

$$\big|\psi\big\rangle_{\text{continuous}} = \sum_x \psi(x)\big|x\big\rangle = \ldots\psi(x')\big|x'\big\rangle + \psi(x'')\big|x''\big\rangle + \psi(x''')\big|x'''\big\rangle + \ldots \quad . \tag{B.26}$$

However, the eigenvalue spectrum $\{x\}$ is an *uncountable* set. We can never enumerate the various x eigenstates with natural numbers as we have for the a_k discrete eigenstates. Luckily, Newton has developed an excellent workaround for us. Written in the notation of Leibniz, the workaround is

$$\big|\psi\big\rangle_{\text{continuous}} = \int_{-\infty}^{\infty} dx'\,\psi(x')\big|x'\big\rangle \quad . \tag{B.27}$$

This workaround is great and useful, but it comes at the expense of the complications we have discussed.

B.1.2 Back to the Translation Operator

Now that we understand the wavefunction, we will continue from (B.9) restated here:

$$\hat{\mathcal{J}}(dx)\big|\psi\big\rangle = \int dx'\,\big|x'\big\rangle\big\langle x'-dx\big|x\big\rangle = \int dx'\,\psi(x'-dx)\big|x'\big\rangle \quad . \tag{B.28}$$

We understand that the dx infinitesimal translation is a different sort of object than the dx' differential of the integration variable. We also understand that $\psi(x')=\langle x'|x\rangle$ is used because $x(x')=\langle x'|x\rangle$ is relatively unsightly. The minus sign in the argument of $\psi(x'-dx)$ seems to reflect translation by $-dx$ rather than by the dx that had been intended. This was a feature inherited by our change of variables in (B.7). Apparently, $\mathcal{J}(-dx)$ is the operator that generates translation by dx:

$$\hat{\mathcal{J}}(-dx)\big|\psi\big\rangle = \int dx'\,\psi(x'+dx)\big|x'\big\rangle \quad . \tag{B.29}$$

We have seen that the c_k expansion coefficients give the probability for finding a_k in a measurement of the discrete observable A as $P_k=|c_k|^2$. In general, the c_k are called probability amplitudes, and the product with the complex conjugate c_k^* gives a real-valued probability. In the continuous case, the probability amplitude is the wavefunction, so we get $P(x)=|\psi(x)|^2dx$ which results in a real-valued probability after it is integrated across some range. Since it has to be integrated, we call the modulus squared of $\psi(x)$

$$\big|\psi(x)\big|^2 = \psi^*(x)\psi(x) \quad , \tag{B.30}$$

a *probability density*. Before we operated with $\hat{\mathcal{J}}$, the wavefunction was $\psi(x)$. After, it was $\psi(x+dx)$ and the probability density was $|\psi(x+dx)|^2$. Evidently, the translation operator $\hat{\mathcal{J}}(-dx)$ has shifted the probability density for finding ψ in some region of space by the amount dx. We have succeeded in implementing the desired physics, but we have not yet obtained the analytical form of $\hat{\mathcal{J}}$. To get there, we will impose more physics.

- If $|x\rangle$ is properly normalized to $\langle x|x\rangle=1$, then the translated state $|x'\rangle$ must maintain the normalization:

$$\langle x'|x'\rangle = \Big[\langle x|\hat{\mathcal{J}}^\dagger\Big]\Big[\hat{\mathcal{J}}|x\rangle\Big] = \langle x|\hat{\mathcal{J}}^\dagger\hat{\mathcal{J}}|x\rangle = 1 \qquad \Longrightarrow \qquad \hat{\mathcal{J}}^\dagger\hat{\mathcal{J}} = 1 \;\; .^1 \tag{B.31}$$

In other words, we require that $\hat{\mathcal{J}}$ is a unitary operator. In general, unitary transformations preserve the norm of a ket.

- Two consecutive translations by Δx_1 and Δx_2 must be equal to a single translation by $\Delta x_1 + \Delta x_2$:

$$\hat{\mathcal{J}}(\Delta x_1)\hat{\mathcal{J}}(\Delta x_2) = \hat{\mathcal{J}}(\Delta x_1 + \Delta x_2) \quad . \tag{B.32}$$

- Translation by Δx_1 and then $-\Delta x_1$ must be the identity operation:

$$\hat{\mathcal{J}}(-\Delta x_1)\hat{\mathcal{J}}(\Delta x_1) = \mathbb{1} \qquad \Longrightarrow \qquad \hat{\mathcal{J}}(-\Delta x_1) = \hat{\mathcal{J}}^{-1}(\Delta x_1) \quad . \tag{B.33}$$

(This follows from (B.32) in the case of $\Delta x_2 = -\Delta x_1$.)

- In the limit of vanishing displacement, the translation operator must reduce to the identity:

$$\lim_{dx\to 0}\hat{\mathcal{J}}(dx) = \mathbb{1} \quad . \tag{B.34}$$

We still don't have an exact picture of the analytical form of $\hat{\mathcal{J}}$ though we have obtained have a detailed view of its physics. To move forward, we supplement these physical requirements with a mathematical requirement that $\hat{\mathcal{J}}(dx)$ should be linear in dx to leading order.

$$\Big|\mathbb{1} - \hat{\mathcal{J}}(dx)\Big| = \mathcal{O}(dx) \quad . \tag{B.35}$$

Now the magic is made to happen with... *an ansatz*! We will guess that the form is

$$\hat{\mathcal{J}}(dx) = 1 - i\hat{K}\,dx \quad , \tag{B.36}$$

for some Hermitian operator $\hat{K}$. It is often taken as a postulate of quantum mechanics that the generator of translations $\hat{K}$ is the momentum operator $\hat{p}$ times a constant.

When developing $\hat{\mathcal{J}}$ in [83], Sakurai and Napolitano proceed with a method by which one is able to deduce that the momentum operator satisfies the ansatz. Their method of Taylor series analysis necessarily introduces some gaps in the mathematical rigor at order $\mathcal{O}(dx^2)$. Ignoring $\mathcal{O}(dx^2)$ terms is perfectly standard in physics, and taking $\hat{K} \propto \hat{p}$ directly as a postulate also introduces a gap in the first principles

[1] Dagger denotes the conjugate transpose. For scalars, this is the ordinary complex conjugate.

approach to understanding where everything comes from. However, all the expressions which follow from the $\hat{K} \propto \hat{p}$ postulate are exact while $\mathcal{O}(dx^2)$ gaps propagate through all the expressions which follow from the method of Taylor series analysis. Because the MCM has some concept of changing levels of aleph such that dx on one level of aleph might be finite on another level, we prefer to stay away from ignoring the $\mathcal{O}(dx^2)$ terms. Therefore, we postulate that $\hat{K}$ is $\hat{p}$ times a constant. From (B.36), we can see that $\hat{K}$ does not have the correct units to be the momentum operator which should have units of mass times velocity. As it is, $\hat{K}$ has units of inverse meters. Dimensional analysis shows that $\hat{p}$ must be divided by something with units of action if it is to play the role of the generator of translations. Sakurai and Napolitano mention that if quantum physics had been developed in history before classical physics, the fundamental units would have been chosen so that this constant of proportionality between $\hat{K}$ and $\hat{p}$ was equal to one [83]. With units already having been set, it works out to $\hbar$:

$$\hat{\mathcal{J}}(dx) = 1 - \frac{i}{\hbar}\,\hat{p}\,dx \quad . \tag{B.37}$$

Finite translations are obtained by compounding infinitesimal ones. To proceed in our quest to obtain the analytical form of

$$\hat{\mathcal{J}}(\Delta x)\big|x\big\rangle = \big|x + \Delta x\big\rangle \quad , \tag{B.38}$$

we divide the finite (non-infinitesimal) translation Δx into N equal parts

$$\delta x = \frac{\Delta x}{N} \quad . \tag{B.39}$$

Applying (B.32), we have

$$\hat{\mathcal{J}}(\Delta x) = \hat{\mathcal{J}}\left(\sum_{k=1}^{N} \delta x\right) = \prod_{k=1}^{N} \hat{\mathcal{J}}(\delta x) \quad . \tag{B.40}$$

We make the connection to the generator of infinitesimal translations by taking the limit $N \to \infty$ such that $\delta x \to dx$. This yields

$$\hat{\mathcal{J}}(\Delta x) = \lim_{N\to\infty} \prod_{k=1}^{N} \hat{\mathcal{J}}(\delta x) = \lim_{N\to\infty} \left(1 - \frac{i\hat{p}_x \Delta x}{\hbar N}\right)^{N} \quad . \tag{B.41}$$

This limit is a definition of the exponential function, so we have

$$\hat{\mathcal{J}}(\Delta x) = \exp\left\{ -\frac{i\hat{p}_x \Delta x}{\hbar} \right\} \quad . \tag{B.42}$$

If we are able to determine the analytical form of $\hat{p}$ operating on position states, then we will have found the analytical form of the translation operator for such states.

B.2 The Momentum Operator

This section begins with a brief account of the road which led to "the creation of quantum mechanics" for which Heisenberg won the 1932 Nobel Prize in Physics. In 1925, Dirac described the kernel of what Heisenberg had done [232].

> "It is well known that the experimental facts of atomic physics necessitate a departure from the classical theory of electrodynamics in the description of atomic phenomena. This departure takes the form, in Bohr's theory, of the special assumptions of the existence of stationary states of an atom, in which it does not radiate,[1] and of certain rules, called quantum conditions, which fix the stationary states and the frequencies of the radiation emitted during transitions between them.[2] These assumptions are quite foreign to the classical theory, but have been very successful in the interpretation of a restricted region of atomic phenomena. The only way in which the classical theory is used is through the assumption that the classical laws hold for the description of the motion in the stationary states, although they fail completely during transitions, and the assumption, called the Correspondence Principle, that the classical theory gives the right results in the limiting case when the action per cycle of the system is large compared to Planck's constant h, and in certain other special cases.
>
> "In a recent paper *[233]* Heisenberg puts forward a new theory, which suggests that it is not the equations of classical mechanics that are in any way at fault, but that the mathematical operations by which physical results

[1]Classical electromagnetic theory predicts that electrons undergoing centripetal acceleration in atomic orbits should radiate energy and fall into the nucleus. All classical charged particles radiate energy and the non-radiation of electrons in atomic orbitals was one of the main non-classical problems in the early days of atomic physics.

[2]In celestial mechanics, it is understood that a large asteroid impact might subtly alter the orbital radius of a planet around the sun. In atomic physics, the situation is totally different. When a photon comes and hits an atomic electron, it cannot alter the electron's orbit slightly. If the photon does not have enough energy to knock the electron all the way to the next fixed stationary state, then the photon will scatter elastically from the electron. This phenomenon describes the nature of quantum mechanics. In celestial mechanics, there are a continuum of orbital radii allowed for a planet to orbit the sun but in the atomic version of the solar system with electrons orbiting nuclei, the electron is only allowed certain discrete, or *quantized*, orbits.

are deduced from them require modification. All the information supplied by the classical theory can thus be made use of in the new theory. [...]

"We are now in a position to perform the ordinary algebraic operations on quantum variables. The sum of [*matrices*] x and y[*, with the nm matrix element of x denoted* $x(nm)$] is determined by the equations

$$\{x+y\}(nm) = x\big(nm\big) + y\big(nm\big) \quad , \tag{B.42}$$

and the product by

$$xy(nm) = \sum_k x(nk)y(km) \quad , \tag{B.43}$$

[...] An important difference now occurs between the two algebras. In general

$$xy(nm) \neq yx(nm) \quad , \tag{B.44}$$

and quantum multiplication is not commutative, although, as is easily verified, it is associative and distributive. The quantity with components $xy(nm)$ [...] we shall call the Heisenberg product of x and y, and shall write simply as xy. Whenever two quantum quantities occur multiplied together, the Heisenberg product will be understood. Ordinary multiplication is, of course, implied in the products of amplitudes and frequencies and other quantities that are related to sets of n's which are explicitly stated."

Quantum quantities are what we now call observable operators. The quantities we observe commute in the usual way, but their representations in quantum theory do not. The principle manifestation of Heisenberg's quantum algebra is the commutator of position and momentum

$$[\hat{x}_j, \hat{p}_k] \equiv \big(\hat{x}_j\hat{p}_k - \hat{p}_k\hat{x}_j\big) = i\hbar\delta_{jk} \qquad \Longrightarrow \qquad \hat{x}\hat{p}_x \neq \hat{p}_x\hat{x} \quad . \tag{B.45}$$

A wonderful feature of quantum mechanics is that observables which can be known simultaneously have operators that commute. If two observables can't be known at the same time, their operators don't commute, meaning the commutator $[\hat{A}, \hat{B}]$ does not vanish. When two operators commute, we are able to find simultaneous eigenstates of both which save us the hassle of change of basis operations each time one or the other observable is to be measured. Intuitively, we know that 3D position can be measured in the lab, so we expect that the $\hat{x}$, $\hat{y}$, and $\hat{z}$ observable operators should commute. $\psi(\mathbf{x}) = \delta(\mathbf{x})$ is a simultaneous eigenstate of all three operators which we denote $|\mathbf{x}\rangle = |x, y, z\rangle$. That position and momentum can't commute follows from similar physical thinking. To measure momentum, one measures speed, mass, and direction. To measure speed, time is measured between two positions. Once a speed is determined, however, one must ask which of the two positions might be associated with it. Since we have only measured speed between two positions, we cannot rightly associate either of them with the measured speed. If we were to associate the average of the two positions with the speed, that would require an assumption of constant velocity between the two positions. This would be unphysical because we measured the average velocity between the two positions and have no way to know if it was constant on the interval. Therefore, we can be sure that $\hat{\mathbf{p}}$ won't commute with $\hat{\mathbf{x}}$ because the underlying quantities cannot be known simultaneously.[1] Other than that, we need to determine the analytical form of $\hat{\mathbf{p}}$ if we are going to answer the previous question about the analytical form of the translation operator $\hat{\mathcal{J}}(\Delta\mathbf{x})$ which depends on it, as in (B.42).

The guiding principle regarding the form of $\hat{\mathbf{p}}$ is that it has to return eigenvalue $\mathbf{p}$ when it operates on a momentum eigenstate. Following along with the goal to determine the form of $\hat{\mathcal{J}}(\Delta\mathbf{x})$ acting states in the position representation, we will consider momentum eigenstates in the position representation. Momentum eigenstates in the momentum representation can only be δ functions,[2] and, since the position representation is the Fourier transform of the momentum representation, the momentum eigenstate in that representation has to be a plane wave. Omitting factors of 2π and $\hbar$, the Fourier transform of $\psi(p) = \delta(p' - p)$ is

$$\psi(x) = \int dp'\, e^{-ip'x}\psi(p') = \int dp'\, e^{-ip'x}\delta(p' - p) = e^{-ipx} \quad . \tag{B.46}$$

Momentum can be to the left or right (p can be positive or negative), so we may ignore the minus sign to write the *matrix elements* of a momentum eigenstate in the position representation as

$$\langle x|p\rangle = \psi_p(x) = e^{ipx/\hbar} \quad . \tag{B.47}$$

If x and p were discrete, it would be easy to see that the set of all $\langle a_j|b_k\rangle$ forms a $j \times k$ matrix. For ease of description, we still call $\langle x|p\rangle$ a matrix element even though x and p are continuous. The subscript on $\psi_p(x)$ tells us that this is the wavefunction of the momentum eigenstate with eigenvalue p. Although momentum

[1] It is somewhat miraculous that quantum theory should contain such physical constraints. By rights, there is no need for a theory to contain this functionality without additional input requiring it.

[2] This detail cannot be glossed over. Anyone attempting to learn QM from this appendix must be absolutely sure that they know exactly why a momentum eigenstate in the momentum representation must be a Dirac δ function.

eigenstates cannot be measured (all we can observe is momentum in some range) Heisenberg uncertainty implies that exact knowledge of momentum implies maximal uncertainty in position. Thus, momentum eigenstates are maximally diffuse plane waves in the position representation. For a given p_0 and x_0, the expression $\langle x|p\rangle$ gives the probability amplitude that a particle with momentum p_0 will be found at position x_0. In other words, $\langle x|p\rangle$ is the wavefunction of the momentum eigenstate. Formally, we might say that there exist normalized solutions to Schrödinger's equation in the form

$$\psi_p(x) = A\exp\left\{\frac{i\big(px - Et\big)}{\hbar}\right\} = c(t)\exp\left\{\frac{ipx}{\hbar}\right\} \quad , \tag{B.48}$$

but it suffices to ignore the time part. By optical inspection of (B.47) or (B.48), one determines that the momentum operator returning eigenvalue p when acting on a momentum eigenstate in the position representation is $-i\hbar\partial_x$:

$$\hat{p}\psi_p(x) = -i\hbar\frac{\partial}{\partial x}\exp\left\{\frac{ipx}{\hbar}\right\} = p\exp\left\{\frac{ipx}{\hbar}\right\} = p\psi_p(x) \quad . \tag{B.49}$$

If we had used the e^{-ipx} wavefunction, then we would have gotten the $-p$ eigenvalue which is correct for a plane wave moving in the other direction. Ultimately, we take it as a postulate of quantum mechanics (surprise!) that the position representation of the momentum operator is

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \quad . \tag{B.50}$$

The Heisenberg algebra follows directly:

$$\begin{aligned}
[\hat{x},\hat{p}]\psi_p &= \hat{x}\hat{p}\psi_p - \hat{p}\hat{x}\psi_p \\
&= -i\hbar\hat{x}\frac{\partial}{\partial x}\psi_p + i\hbar\frac{\partial}{\partial x}\big(x\psi_p\big) \\
&= xp\psi_p + \left(i\hbar\psi_p + i\hbar x\frac{\partial}{\partial x}\psi_p\right) \\
&= xp\psi_p + \big(i\hbar\psi_p - xp\psi_p\big) \\
&= i\hbar\psi_p \quad .
\end{aligned} \tag{B.51}$$

Now we may plug $\hat{p}$ into (B.42) to write

$$\hat{\mathcal{J}}(\Delta x) = \exp\left\{-\frac{i\hat{p}_x\Delta x}{\hbar}\right\} = \exp\left\{-\Delta x\frac{\partial}{\partial x}\right\} \quad , \tag{B.52}$$

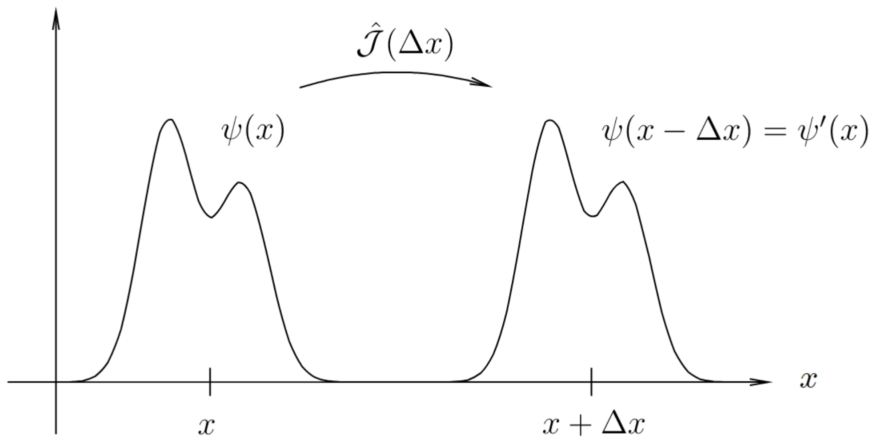

Figure 31: This figure adapted from Littlejohn [234] shows the action of the translation operator on an arbitrary wavefunction in the position representation. While the translation application from, say, $\mathcal{H}$ at x to Ω at $x + \Delta x$ is obvious, more complicated operations are required for MCM applications. The MCM operation must alter the shape of $\psi(x)$. Such operations are time evolutions rather than spatial translations.

which is the analytical representation of $\hat{\mathcal{J}}$! Testing it on the ψ_p wavefunction yields

$$\begin{aligned}
\hat{\mathcal{J}}(\Delta x)\psi_p(x) &= \exp\left\{-\Delta x \frac{\partial}{\partial x}\right\} \exp\left\{\frac{ipx}{\hbar}\right\} \\
&= \exp\left\{\frac{-ip\Delta x}{\hbar}\right\} \exp\left\{\frac{ipx}{\hbar}\right\} \\
&= \exp\left\{\frac{ip\big(x - \Delta x\big)}{\hbar}\right\} \\
&= \psi_p(x - \Delta x) \quad .
\end{aligned} \tag{B.53}$$

This agrees with (B.29). Another way to understand what is going on is to write

$$\hat{\mathcal{J}}(\Delta x)\psi_p(x) = \psi_p'(x) \quad , \qquad \text{and} \qquad \psi_p'(x + \Delta x) = \psi(x) \quad . \tag{B.54}$$

This tells us that the translated wavefunction ψ_p' at the shifted position is equal to the original wavefunction ψ at the unshifted position, as in Figure 31.

Now that we know what the momentum operator is, we may proceed with the derivation of the momentum operator as the generator of translations. We previously skipped this around (B.36) by assuming (postulating)

$$\hat{\mathcal{J}}(dx) = 1 - i\hat{K}\,dx \quad , \quad \text{and} \qquad \hat{K} \propto \hat{p} \quad , \tag{B.55}$$

but now we will derive the $\hat{K} \propto \hat{p}$ part of our assumption. Translated by some small amount, the momentum eigenfunction is

$$\hat{\mathcal{J}}(-\delta x)\psi_p(x) = \psi_p(x+\delta x) = \exp\big\{ip\big(x+\delta x\big)\big\} = e^{ip\delta x}e^{ipx} = e^{ip\delta x}\psi_p(x) \quad . \tag{B.56}$$

We expand the displacement term as

$$\psi_p(x+\delta x) = \big[1 + ip\delta x + \mathcal{O}(\delta x^2)\big]\,\psi_p(x) \quad , \tag{B.57}$$

and compare to the Taylor series expansion of $\psi_p(x+\delta x)$ around x:

$$\begin{aligned}\psi_p(x+\delta x) &= \psi(x)_p + \delta x\frac{d}{dx}\psi_p(x) + ... \\ &= \left[1 + i\delta x\left(-i\frac{d}{dx}\right) + ...\right]\psi_p(x) \quad .\end{aligned} \tag{B.58}$$

Equating $\mathcal{O}(\delta x)$ terms between (B.57) and (B.58), we find

$$-i\frac{d}{dx}\psi_p(x) = p\psi_p(x) \quad . \tag{B.59}$$

This confirms that we have the correct form for the momentum operator. In the limit of infinitesimal δx, we ignore the $\mathcal{O}(\delta x^2)$ part of (B.57) to write

$$\begin{aligned}\psi_p(x+\delta x) &= (1 + ip\delta x)\,\psi_p(x) \\ &= \left[1 + i\delta x\left(-i\frac{d}{dx}\right)\right]\psi_p(x) \\ &= (1 + i\hat{p}\delta x)\,\psi_p(x) \\ &= \Big[1 - i\hat{K}\big(-\delta x\big)\Big]\,\psi_p(x) \\ &= \hat{\mathcal{J}}(-\delta x)\psi_p(x) \quad .\end{aligned} \tag{B.60}$$

By ignoring the $\mathcal{O}(\delta x^2)$ terms and assuming that we can set terms of equal order in δx equal between (B.57) and (B.58), and by assuming $\hat{\mathcal{J}} = 1 - i\hat{K}\,dx$ to begin with, we have made a derivation showing that the momentum operator is the generator of spatial translations. This supplements our postulate/axiom which says the same thing.

Momentum in quantum mechanics goes on to be very complicated. Mainly, it is only possible to define the momentum operator as the mass times the derivative of position when the vector potential $\mathbf{A}$ is equal to zero. This equality gives what is called the canonical momentum operator $\hat{\mathbf{p}}$. In general, however, we have

$$\frac{d}{dt}\hat{\mathbf{x}} = \frac{1}{m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\hat{\mathbf{x}})\right) \quad . \tag{B.61}$$

Thus, we introduce the kinematical momentum operator

$$\hat{\mathbf{\Pi}} = m\frac{d}{dt}\hat{\mathbf{x}} = \hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\hat{\mathbf{x}}) \quad . \tag{B.62}$$

The main difference between the canonical and kinematical momenta is

$$[\hat{p}_k, \hat{p}_j] = 0 \quad , \quad \text{while} \qquad [\hat{\Pi}_k, \hat{\Pi}_j] \neq 0 \quad . \tag{B.63}$$

It is known that the vector potential is not unique, and the tricks that one can play with $\mathbf{A}(\hat{x})$ are the main inroads to theories of gauge freedom, or gauge theories. Usually the choice of one $\mathbf{A}(\hat{x})$ or another is called *fixing the gauge*. In turn, this defines the kinematical momentum operator which replaces the $\hat{p}$ we have postulated above.

B.3 The Time Evolution Operator

It is said that time doesn't exist in quantum mechanics. What is meant is that states in Hilbert space are represented as functions of spatial variables but not time. The time dependence is added to states as a phase factor which is constant in the Hilbert space of states at time t. In the case of time-dependent Hamiltonian operators, there is a Hilbert space of energy eigenstates corresponding to every possible $\hat{H}(t)$. Even in the case of a time-independent Hamiltonian, there is still a Hilbert space of energy eigenstates at every time t. This can get glossed over since the eigenstates in each Hilbert space are the same complete set of orthonormal basis states. However, the eigenstates of every observable operator do, in fact, belong to a Hilbert space at a specific time which is distinct from the space of states at any other time.

To develop time evolution, we will introduce the symbol $|\psi, t_0; t\rangle$ as the state of a system at time $t > t_0$ that was already observed to be in state ψ at time t_0. We have previously implemented the spatial translation operator $\hat{\mathcal{J}}$ such that

$$\hat{\mathcal{J}}(\Delta\mathbf{x})\big|\mathbf{x}\big\rangle = \big|\mathbf{x}+\Delta\mathbf{x}\big\rangle \quad , \tag{B.61}$$

and now we will develop the time translation operator as

$$\hat{\mathcal{U}}(t,t_0)\big|\psi,t_0\big\rangle = \big|\psi,t_0;t\big\rangle \quad . \tag{B.62}$$

By convention, this is called the time evolution operator. The added argument t_0 tells us that $\hat{\mathcal{U}}(t,t_0)$ *only* operates on the Hilbert space of states which exist at time t_0. This is redundant for time-independent Hamiltonians, but it is not redundant in general. The requirements imposed on $\hat{\mathcal{U}}(t,t_0)$ are mostly the same as those imposed on $\hat{\mathcal{J}}$.

- If $|\psi,t_0\rangle$ is properly normalized to $\langle\psi,t_0|\psi,t_0\rangle = 1$, then the time evolved state $|\psi,t_0\rangle$ must maintain the normalization:

$$\big\langle\psi,t_0;t\big|\psi,t_0;t\big\rangle = \big\langle\psi,t_0\big|\hat{\mathcal{U}}^\dagger\hat{\mathcal{U}}\big|\psi,t_0\big\rangle = 1 \qquad \Longrightarrow \qquad \hat{\mathcal{U}}^\dagger(t_0,t)\hat{\mathcal{U}}(t_0,t) = 1 \quad . \tag{B.63}$$

- Two consecutive time evolutions, $\hat{\mathcal{U}}(t_1,t_0)$ followed by $\hat{\mathcal{U}}(t_2,t_1)$, must be equal to a single time evolution by $\hat{\mathcal{U}}(t_2,t_0)$:

$$\hat{\mathcal{U}}(t_2,t_1)\hat{\mathcal{U}}(t_1,t_0) = \hat{\mathcal{U}}(t_2,t_0) \quad . \tag{B.64}$$

- Evolution by Δt_1 and then $-\Delta t_1$ must be the identity operation:

$$\hat{\mathcal{U}}(-\Delta t_1)\hat{\mathcal{U}}(\Delta t_1) = \mathbb{1} \qquad \Longrightarrow \qquad \hat{\mathcal{U}}(-\Delta t_1) = \hat{\mathcal{U}}^{-1}(\Delta t_1) \quad . \tag{B.65}$$

- In the limit of vanishing temporal displacement, the evolution operator must reduce to the identity:

$$\lim_{t\to t_0}\hat{\mathcal{U}}(t,t_0) = \mathbb{1} \quad . \tag{B.66}$$

- $\hat{\mathcal{U}}(t_0+dt,t_0)$ should be linear in dt to leading order:

$$\big|1-\hat{\mathcal{U}}(t_0+dt,t_0)\big| = \mathcal{O}(dt) \quad . \tag{B.67}$$

The unitarity condition of (B.63) is required for the preservation of the probability interpretation in which $c_k^2(t)$ is the probability for finding eigenvalue a_k at time t. This is demonstrated when we require that the sum of the squares of the expansion coefficients in a particular basis must sum to unity at all times. To demonstrate, we expand in the discrete basis of $|a_k\rangle$:

$$\big|\psi,t_0\big\rangle=\sum_k\big|a_k\big\rangle\big\langle a_k\big|\psi,t_0\big\rangle=\sum_k c_k(t_0)\big|a_k\big\rangle\quad . \tag{B.68}$$

The meaning of $c_k(t_0)$ is exactly the same as the previous meaning of c_k. We add the time dependence because the probability for finding the a_k eigenvalue in a measurement on $|\psi\rangle$ might not be constant in time. For example, if one prepares a system in an excited state, it will become less and less likely that one will observe the system in the excited state as time goes on. On long time scales, systems tend to return to the ground state and/or come to thermodynamic equilibrium.

Assume that ψ is normalized at t_0. Multiplying (B.68) from the left with $\langle\psi,t_0|$ yields

$$\big\langle\psi,t_0\big|\psi,t_0\big\rangle=\sum_k c_k(t_0)\big\langle\psi,t_0\big|a_k\big\rangle=\sum_k c_k(t_0)c_k^*(t_0)=\sum_k\big|c_k(t_0)\big|^2=1\quad . \tag{B.69}$$

Since t_0 is an arbitrary time, this has to hold for any $t\neq t_0$, so

$$\big\langle\psi,t_0;t\big|\psi,t_0;t\big\rangle=\sum_k\big\langle\psi,t_0;t\big|a_k\big\rangle\big\langle a_k\big|\psi,t_0;t\big\rangle=\sum_k c_k(t)c_k^*(t)=\sum_k\big|c_k(t)\big|^2=1\quad . \tag{B.70}$$

Time evolution can alter the expansion coefficients in the expansion of an abstract state in a certain basis, but the sum the coefficients' absolute squares always adds up to one. This tells us that the probability of finding the state in one of the possible eigenstates is always 100%.

As with the generator of translation $\hat{\mathcal{J}}$, we will assume

$$\hat{\mathcal{U}}(t_0+dt,t_0)=1-i\hat{\Omega}\,dt\quad , \tag{B.71}$$

and then proceed to determine $\hat{\Omega}$. Studying $\hat{\mathcal{J}}$, it was not mentioned that these ansatzes are not exactly unitary. Presently, we have

$$\big(1-i\hat{\Omega}\,dt\big)^\dagger\big(1-i\hat{\Omega}\,dt\big)=1+\hat{\Omega}^2dt^2\quad , \tag{B.72}$$

though unitary operators satisfy

$$\hat{\mathsf{O}}^\dagger\hat{\mathsf{O}}=\mathbb{1}\quad . \tag{B.73}$$

As is usual, we ignore $\mathcal{O}(dt^2)$ terms and proceed via the minimal hand waving method to call $\hat{\mathcal{U}}$ the *unitary time evolution operator*. There are some principles of classical mechanics which motivate the Hamiltonian as the generator of time evolutions, but we will simply postulate

$$\hat{\Omega} = \frac{1}{\hbar}\hat{H} \quad . \tag{B.74}$$

The Hamiltonian operator $\hat{H}$ is constructed by promoting all instances of positions and momenta in the classical Hamiltonian to their corresponding operators, or "quantum quantities."

Now we will derive the fundamental equation for $\hat{\mathcal{U}}$. The composition property of $\hat{\mathcal{U}}$ is given by (B.64). Combining the compositive law with the (B.71) ansatz, we have

$$\begin{aligned}\hat{\mathcal{U}}(t+\delta t, t_0) &= \hat{\mathcal{U}}(t+\delta t, t)\,\hat{\mathcal{U}}(t, t_0) \\ &= \left(1 - i\frac{1}{\hbar}\hat{H}\,\delta t\right)\hat{\mathcal{U}}(t, t_0) \\ &= \hat{\mathcal{U}}(t, t_0) + \frac{1}{i\hbar}\hat{H}\,\delta t\,\hat{\mathcal{U}}(t, t_0) \quad .\end{aligned} \tag{B.75}$$

By moving $\hat{\mathcal{U}}(t, t_0)$ to the left hand side and multiplying both sides by $i\hbar/\delta t$, we obtain

$$i\hbar\,\frac{\hat{\mathcal{U}}(t+\delta t, t_0) - \hat{\mathcal{U}}(t, t_0)}{\delta t} = \hat{H}\,\hat{\mathcal{U}}(t, t_0) \quad . \tag{B.76}$$

In the limit $\delta t \to dt$, the left side contains the definition of the derivative with respect to t:

$$i\hbar\,\frac{\partial}{\partial t}\hat{\mathcal{U}}(t, t_0) = \hat{H}\,\hat{\mathcal{U}}(t, t_0) \quad . \tag{B.77}$$

As it turns out, (B.77) is the Schrödinger equation for the time evolution operator. We obtain Schrödinger's equation for states by multiplying from the right with $|\psi, t_0\rangle$. This yields

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\hat{\mathcal{U}}(t, t_0)\big|\psi, t_0\big\rangle &= \hat{H}\,\hat{\mathcal{U}}(t, t_0)\big|\psi, t_0\big\rangle \\ i\hbar\frac{\partial}{\partial t}\big|\psi, t_0; t\big\rangle &= \hat{H}\big|\psi, t_0; t\big\rangle \quad ,\end{aligned} \tag{B.78}$$

which is the famous time-dependent Schrödinger equation. If we know how $\hat{\mathcal{U}}(t, t_0)$ evolves, then we don't need Schrödinger's equation for states. We can operate directly on the states with the time evolution operator $\hat{\mathcal{U}}$ to generate states at arbitrary times given that we know the state at t_0. Therefore, we will solve Schrödinger's equation for $\hat{\mathcal{U}}(t,t_0)$, and then act on states with $\hat{\mathcal{U}}$ to obtain states at later times.

First we will examine the time-independent case of $\hat{H}\neq\hat{H}(t)$. The familiar looking (hopefully) differential equation (B.77) is solved by optical inspection as

$$\frac{\partial}{\partial t}\hat{\mathcal{U}}(t,t_0)=\frac{1}{i\hbar}\hat{H}\,\hat{\mathcal{U}}(t,t_0)\qquad\Longrightarrow\qquad\hat{\mathcal{U}}(t,t_0)=\exp\left\{\frac{-\,i\hat{H}\big(t-t_0\big)}{\hbar}\right\}\quad.\tag{B.79}$$

As a reminder that not all differential equations are solved by optical inspection, we continue from (B.77) as

$$\begin{aligned}\frac{\partial}{\partial t}\hat{\mathcal{U}}(t,t_0)&=\frac{1}{i\hbar}\hat{H}\,\hat{\mathcal{U}}(t,t_0)\\ \frac{1}{\hat{\mathcal{U}}(t,t_0)}\frac{\partial}{\partial t}\hat{\mathcal{U}}(t,t_0)&=\frac{1}{i\hbar}\hat{H}\\ \int_{t_0}^{t}dt'\,\frac{1}{\hat{\mathcal{U}}(t',t_0)}\frac{\partial}{\partial t'}\hat{\mathcal{U}}(t',t_0)&=\frac{1}{i\hbar}\hat{H}\int_{t_0}^{t}dt'\quad.\end{aligned}\tag{B.80}$$

We proceed by u-substitution:

$$u=\hat{\mathcal{U}}(t',t_0)\qquad\Longrightarrow\qquad du=\frac{\partial}{\partial t'}\hat{\mathcal{U}}(t',t_0)dt'\quad,\tag{B.81}$$

yields

$$\int_{u(t_0)}^{u(t)}\frac{du}{u}=\int_{\hat{\mathcal{U}}(t_0,t_0)}^{\hat{\mathcal{U}}(t,t_0)}\frac{d\hat{\mathcal{U}}}{\hat{\mathcal{U}}}\quad.\tag{B.82}$$

We continue from (B.80) as

$$\begin{aligned}\int_{\hat{\mathcal{U}}(t_0,t_0)}^{\hat{\mathcal{U}}(t,t_0)}\frac{d\hat{\mathcal{U}}}{\hat{\mathcal{U}}}&=\frac{1}{i\hbar}\hat{H}\int_{t_0}^{t}dt'\\ \ln\hat{\mathcal{U}}\,\Bigg|_{\hat{\mathcal{U}}(t_0,t_0)}^{\hat{\mathcal{U}}(t,t_0)}&=\frac{1}{i\hbar}\hat{H}t'\Bigg|_{t_0}^{t}\\ \ln\Big[\hat{\mathcal{U}}(t,t_0)\Big]-\ln\Big[\hat{\mathcal{U}}(t_0,t_0)\Big]&=\frac{1}{i\hbar}\hat{H}\big(t-t_0\big)\quad.\end{aligned}\tag{B.83}$$

The $\hat{\mathcal{U}}(t_0,t_0)$ operator on the left is the identity by (B.66). The log of the identity vanishes. Taking the exponential of both sides yields

$$\hat{\mathcal{U}}(t,t_0) = \exp\left\{\frac{-i\hat{H}\big(t-t_0\big)}{\hbar}\right\} \quad . \tag{B.84}$$

This is the unitary evolution operator for a state at time t_0 subject to a time-independent Hamiltonian. Although the ansatz stated in (B.71) was not exactly unitary, the present form of $\hat{\mathcal{U}}$ given in (B.84) is exactly unitary because it is the exponential of a Hermitian operator.

If the Hamiltonian is a function of time, and if $[\hat{H}(t_1), \hat{H}(t_2)] = 0$ for any t_1, t_2, the solution proceeds identically except we cannot take $\hat{H}(t)$ out of the integral as we have in (B.80). The result is

$$\hat{\mathcal{U}}(t,t_0) = \exp\left\{-\frac{i}{\hbar}\int dt'\,\hat{H}(t')\right\} \quad . \tag{B.85}$$

If the Hamiltonian is a function of time and $[\hat{H}(t_1), \hat{H}(t_2)] \neq 0$, then the solution is much more complicated. In general, it will be expressed as a Dyson series:

$$\hat{\mathcal{U}}(t,t_0) = 1 + \sum_{k=1}^{\infty}\left(\frac{-i}{\hbar}\right)^k \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \,...\int_{t_0}^{t_{k-1}} dt_n\,\hat{H}(t_1)\hat{H}(t_2.)..\hat{H}(t_k) \quad . \tag{B.86}$$

The $k = 1$ term is a single integral, the $k = 2$ term is a double integral, etc. The $k = \infty$ term is an infinite-dimensional integral, notably. Although any finite $\Delta t = t - t_0$ necessarily contains an uncountable infinity of different times t_k at which the Hamiltonian does not commute, the countable terms of the Dyson series offer a decent approximation. Examples of the three increasingly difficult cases of $\hat{\mathcal{U}}$ are a spin magnetic moment in (i) a static field such that $\hat{H} \neq \hat{H}(t)$, (ii) a time varying field with a constant direction such that $\hat{H} = \hat{H}(t)$ but $[\hat{H}(t_1), \hat{H}(t_2)] = 0$, and (iii) a field varying in strength and direction such that the time-dependent Hamiltonians at different times do not commute.

Observables that commute with the Hamiltonian are constants of the time evolution generated by Schrödinger's equation. In general, one defines a *correlation amplitude* $C(t)$ as a measure of the difference between $|\psi, t_0\rangle$ and $|\psi, t_0; t\rangle$. $C(t)$ is a measure of how quickly diffusion sets in, or how quickly a state will thermalize. Thermalization is the process by which an eigenstate will evolve into a superposition of eigenstates if left unobserved.

This appendix has described what is called the Schrödinger picture of quantum mechanics, but there exist other pictures such as the Heisenberg and interaction pictures wherein the conventions for grouping different objects are slightly different. In the Heisenberg picture, for example, operators vary in time while states are constant. A good understanding of the basics requires understanding at least the Heisenberg and Schrödinger pictures.

Appendix C: Historical Context

This appendix first appeared as Section 3.2 in [1]. A few new footnotes and minor edits appear.

The first written description of the MCM appears in [31]. It was rejected by arXiv in September 2009, and it is the likely basis for the articles titled "Is the Universe Inside a Black Hole?" that Nikodem Poplawski has been successfully pushing to popular media since 2010 [235–243]. The MCM phrase *inverse radial spaghettification*[1] [39] is a fancy way to say that the universe is inside a black hole. In newer research, we have gone on to show that the observer resides on a singularity at the origin of coordinates marking each level of aleph.[2] It is commonly understood that singularities mark the center of black holes, so universe-in-a-black-hole is very much a facet of the MCM. We suggest that Poplawski began providing material for these articles after he was inspired to do so by the original MCM manuscript [31], which he obtained *somehow*.

At the end of September 2009, similarly, Ashtekar, Campiglia, and Henderson published [49] wherein the first citation is to the Feynman paper [67] that was considered in the introduction to [1]. This is interesting because Ashtekar had not been citing Feynman's war era papers from 70 years earlier, but then he did do so immediately after this writer distributed [31]. That paper began with a quote taken from one of Feynman's less famous war era papers where he makes a comment about the time ordering of events not being as important as the way events are encoded in his formalism. ArXiv lists the submission date on Ashtekar et al.'s paper [49] as about one or two weeks after an anonymous reviewer at arXiv rejected [31].[3] Since LQC was multiply cited in [31][4]—LQC being a theory whose bottom-liners include Ashtekar[5]—it is not unlikely that the arXiv reviewer, if that was not Ashtekar himself, sent the manuscript to Ashtekar.

Ashtekar may have obtained the manuscript not through arXiv but through another channel. Just weeks before Ashtekar et al. published [49], this writer had distributed [31] in the newly opened Center for Relativistic Astrophysics (CRA) at Georgia Tech whose founding faculty include two former colleagues of Ashtekar's:

[1] Inverse radial spaghettification, a term coined in [39], describes the MCM mechanism for dark energy dependent on the rarefaction of time as the present accelerates toward the future more quickly than the past. Time rarefaction was called *inverse radial spaghettification.*

[2] We have since sought to disassociate the present and the singularity as $\mathcal{H}$ and $\varnothing$. However, this convention was in place during the main publication period for the Poplawski articles [235–243].

[3] Unfortunately, we have no record of the date of the original submission of [31] to arXiv. It was probably around September 15, 2009.

[4] LQC and LQG were not cited directly in 2009. Instead we used the terms "bouncing" and "the repulsive force of quantum geometry" which were taken from Ashtekar's 2009 LQC talk at Georgia Tech. (The record of this talk was subsequently deleted from the internet.)

[5] The bottom-liners also include Bojowald who declared LQC "dead" in 2013 (see [110]).

Pablo Laguna[1] and Deirdre Shoemaker. The purpose of the email distribution was to advertise that this writer would give a talk on the MCM in the CRA that week. Shoemaker, who had been working side by side with Ashtekar in Pennsylvania just a year earlier, attended the talk, but she was *most intently* on her phone for the duration,[2] almost intentionally projecting disinterest, or *disrespect*, and is unlikely to have made any effort to help this writer disseminate his research.

The key point in all of this is that somehow [31] was deemed not good enough even to appear on arXiv as a preprint though it was good enough to prompt *an immediate response paper* from leading names in the field [49]. Usually eliciting a response paper at all is considered a high achievement in theoretical physics, and an immediate response from a leader in the field (Ashtekar) is high praise indeed.[3] As a counterexample, consider that many papers passing the "very high," "very meaningful," and "critically important" bar of peer review go on to be completely ignored and accumulate a layer of dust serving as a reminder that it did, at one point, pass peer review, meaning that the publishing cartel bestowed a cookie upon the authors who can all add the cookie crumbles to their CV's... which mean nothing weighed against the merit of the research that appeared in the publication. The cartel's cookie crumbles have become overly important in the modern era where the merit of the research in question is too often non-existent or not significant.

Despite science's alleged self-correcting mechanism, the exact dynamic from 2009 unfolded again in 2011. Once again, arXiv rejected another manuscript [39] based on their unpublished, uncited censorship guidelines. It seems that after this later manuscript made the back channel rounds, negative frequency resonant radiation was immediately discovered [42], and a team at USC immediately built a working quantum computer [244]. Note that since frequency is inverse time, negative frequency resonant radiation is a negative time mode exactly like the $|t_-\rangle$ state suggested only months earlier in [39]. In [30], we suggested to look for correlations with delay, and then the BaBar collaboration announced that they had decided to reanalyze their old data for correlations with delay, and that they did affirmatively find them [32] just a few months later.

In 2009, the first account of the MCM [31] was not even good enough to be allowed as an arXiv preprint, but it garnered a praiseworthy response. In 2011, [39] still

[1] Laguna deserves an honorable mention and thanks for inviting not just Ashtekar to Georgia Tech, but also Penrose, meaning that both of the speakers that inspired the MCM were the invitees of Laguna.

[2] One wonders how Shoemaker could pursue a PhD, make it through the academic grinder into a tenure track position, get a promotion as a founding member of a center for relativistic astrophysics, and then show absolutely no interest when some of the most important astrophysical mysteries of the universe are plainly spelled out before her eyes on a whiteboard. Affirmative action likely explains the whole thing.

[3] Finkelstein wrote two MCM response papers [146, 147] after arXiv rejected [31], but before they rejected [39].

did not meet the bar of arXiv's unpublished censorship criteria. Not only did that update garner MCM response papers, it garnered MCM response experiments. This is high praise indeed because experiments cost time *and* money whereas papers only cost time. It means that the "peers" of this writer have "reviewed" the manuscript and decided to change research direction in favor of the MCM. If the results of the experimental response had been negative, then the praise would be lessened only somewhat because it would still be true that we had presented an admirable new idea. This is the primary function of theorists: to theorize new theories. In that regard, one may compare the MCM to other very famous theories that are worse and yet still manage to reap all of the praise offered by the community of theorists. Unlike the experimental tests of most respected and praiseworthy theories, however, the results of the MCM experimental response were all affirmative. Therefore, although the MCM has not passed "peer review," it has been known for an experimental fact—multiple experimental facts actually [32, 42, 244] (*at least!*)—that it describes Nature better than any other theory that currently exists. This was known all throughout 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, and at least several months in 2022, but there has been no accompanying update to the public understanding of science.

We are essentially accusing Abhay Ashtekar, Nikodem Poplawski, and others of plagiarism but in the technical sense there has been no plagiarism. In the technical sense, the complaints listed here only suggest that the alleged self-correcting mechanism in science does not exist, and many tenured professionals do not conform to certain ethical standards. We pointed out Ashtekar et al.'s spurious Feynman citation as evidence of his having viewed [31], so consider that in [49], Ashtekar et al. wrote that they were being so vague not to avoid writing about the MCM directly, but rather because they would leave "the detailed derivations and discussions to a longer article." Did those derivations exist at the time of the publication of [49]? Had they been first suggested after someone looked at the 2009 manuscript which arXiv rejected [31]? Perhaps they were suggested but not carried out during the hasty preparation and revision of the rough draft that preceded the preprint cited here as [49]? Perhaps the hastiness in that regard was motivated by a desire to fabricate a parallel false genesis for what *very obviously* appeared to them to be a fantastic new idea? One wonders if the promised detailed derivations ever did appear in the literature. If not, did they ever come into existence? If not, was [49] worded so as to mislead readers about the existence of the derivations?

Ashtekar et al. write the following in [49]. However, one wonders how they managed to report a "rigorously developed Hamiltonian theory" without reporting a rigorous development of anything at all. To the extent that MCM papers are sometimes said to contain "nonsense," it is suggested that this excerpt from [49] contains nonsense.

> "Because of [...] the Schrödinger equation we can now pass to a sum over histories a la Feynman. [...] We emphasize that the result was derived from a Hamiltonian theory. We did not postulate that [*our equation*] is given by a formal path integral. Rather a rigorously developed Hamiltonian theory guaranteed that [*our equation*] is well-defined."

In 2009's [31], we did not include a detailed derivation, and we did not claim rigor without derivation which is what Ashtekar et al. have done. The diagrams in [31] explain an idea much more clearly than Ashtekar et al. were able to explain anything with their non-rigorous rigor of math salad in [49]. They included neither diagrams nor derivations, but, somehow, their paper was good, and ours was found to be terrible. Not just terrible, [31] was determined to be so unacceptably terrible that it attained the rare bar of rejection at arXiv.

How have Ashtekar et al. "rigorously developed" their theory while leaving the "detailed derivations and discussions to a longer article?" The reader should be very careful to note that if the rigor of Ashtekar et al.'s result is offloaded elsewhere beyond their paper's pages, then [31] and [49] are similar indeed! Ashtekar et al.'s murky, imprecise, arguably self-contradictory wording contrasts [31] wherein the abstract states, "No attempt at quantification is made." Instead, we pursue a qualitative analysis of the diagrams that guarantee our framework is well-defined. This *sharply* contrasts Ashtekar et al.'s [49] when the qualitative discussion of diagrams is practical to a degree far beyond the qualitative analysis of quantitative equations that don't, when taken all together, form a rigorous derivation of anything. Generally, quantitative analysis is only superior to qualitative analysis when it is rigorous. Otherwise, math salad is not as good as pictures.[1]

As an example of real quantitative rigor, consider the unassailable truth of the appearance of the coefficient of Einstein's equation 8π in the first intuitive manipulations of the MCM once the equally unassailable truth of

$$2\pi + \left(\Phi\pi\right)^3 \approx 137 \quad , \tag{C.1}$$

was established. Somehow, certain individuals have slunk into the halls of power in scholarship to convince everyone that Feynman was wrong when he is famously

[1] In [245], Pugh writes, "One thing you will observe about all [*the books I suggest*]—they use pictures to convey the mathematical ideas. Beware of books that don't."

paraphrased as stating that all good physicists have the fine structure constant on the wall in their offices and ask themselves where it comes from, and that no one has a good explanation for it, and that it would probably be related to π if they did.

Given that Ashtekar et al. were able to produce the inferior analysis that became [49] within what was likely just days of reading about the MCM, and all within the context of their own years or decades long familiarity with their own material, it is demonstrated exactly how well-defined the MCM already was in 2009. Ashtekar et al. strongly emphasize that their result was derived from a Hamiltonian theory, but they do not say whether or not they were inspired to make that derivation for the first time immediately after viewing the contentious paper that arXiv rejected in 2009 [31]. To the knowledge of this writer, they have not shown that the claimed derivations exist at all. When they wrote that they did not postulate that their formula was given by a formal path integral, was that to distinguish their paper from [31] wherein we postulated that the MCM is given by the formal path integral?

The critical reader will notice that "detailed derivations and discussions" are left out in both [31] and [49], but only one of them appears on arXiv today. In the acknowledgments section at the end of [49], Ashtekar et al.'s first thanks are to Jerzy Lewandowski who was the advisor or colleague of Poplawski at the University of Warsaw. In April 2010, around the time Poplawski began appearing in very many popular science articles about the universe being in a black hole, Poplawski also published [246]. Note how the title of that paper is evocative of the idea of inverse radial spaghettification:[1] "Radial Motion into an Einstein–Rosen Bridge." Likewise, the title of Lewandowski's October 2009 talk at LSU was evocative: "Spin foams from loop quantum gravity perspective." What was this new perspective that Lewandowski was evangelizing in Louisiana just a month after arXiv rejected [31]?

While on the topic of the conduct of science in a manner that is other than ethical, consider the following. At some point in 2011 while preparing a draft of [39], this writer encountered a slideshow from another a talk given at LSU. The title was something like "Path Integral Approach to Spin Foams," and the name on the slides was likely Jonathan Engle (a speaker in [110]: the "eulogy" for LQC.) The slides were dated from the end of 2008, but when this writer checked on the seminar schedule at the host university, LSU, the talk was really given at the end of 2009. The date from 2008 does not appear to have been "a typo," in the opinion of this writer.

[1]The term "inverse radial spaghettification" did not appear in the literature until 2012 because arXiv did not allow it to be added to the literature in 2011. To understand how the title of Poplawski's 2010 paper is evocative of 2009's [31], note that radial motion means 1D motion, and together with "into an Einstein–Rosen bridge," it means motion toward a bridge between two distant regions of the universe along the 1D manifold defined by the motion. The idea presented in [31] was that dark energy is an expected feature in pairs of worldsheets in the cosmological lattice connected in 1D through a bounce. The connection is 1D because it is along χ^5.

The erroneous time stamp is notable because the path integral formulation of spin foams was not yet conceived in 2008, and a lesser error might not have changed the year of initial formulation to precede the MCM's 2009 path integral cosmology [31]. Based on the description of a new use for the Feynman path integral in [31], and on the fact that Engle was Ashtekar's PhD student, it is likely that the new topic presented and misdated in this talk was inspired by [31]. When one views the LSU Physics and Astronomy talk schedule archives [247], one sees all the years 2004–present, except 2009–2012: the window in which Engle presented the misdated slides. If other researchers were already jockeying in 2009 to position themselves to receive credit for a discovery that was not their own, then whose discovery was it? A full forensic accounting of the failure of physics to self-correct in this regard is required.

Finally, we wish to point out that Lewandowski is a coauthor on [248] which was published in September 2009 around the same time we were proposing to wrap the Minkowski diagram around a cylinder [31]. Therein, Kamiński et al. refer to an unusual cylindrical object $\mathrm{Cyl}(\mathcal{A}(\Sigma))$. One sees that same object in at least one earlier arXiv preprint coauthored by Lewandowski [249], but one wonders if perhaps they have done a more professional time stamp alteration job than was suggested above when discussing Engle's "Path Integral Formulation of Spin Foams" slides.

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