

Relay. Let n be a positive integer and C_i a family of sectors for $i \in \{1, \dots, n\}$. Let $(T)_{ij}: C_i \rightarrow C_j$ be a matrix of calcs, the travel matrix. Let $R: \{(i, c): i \in \{1, \dots, n\} \wedge c \in C_i\} \rightarrow \{1, \dots, n\}$ be a map, the route function. Define the *relay* sequence $(i_k, c_k, T_k)_{k=0}^\infty$ started at $(i_0, c_0) \in \text{dom } R$ with travel matrix $T = T_0$ and route function R as follows: for non-negative integers k , it consists of indices i_k , elements $c_k \in C_{i_k}$, and travel matrices T_k such that, for $i = i_k$ and $j = i_{k+1}$, we have $i_{k+1} = R(i, c_k)$, $c_{k+1} = (T_k)_{ij}(c_k)$,

- $(T_{k+1})_{ij} = (T_k)_{ij}^{c_n}$, and
- $(T_{k+1})_{i'j'} = (T_k)_{i'j'}$ for $(i', j') \neq (i, j)$.

Note that these conditions imply that $(i_{k+1}, c_{k+1}) \in \text{dom } R$ when $(i_k, c_k) \in \text{dom } R$, so by induction it holds for all natural numbers k .

Inuitively, (i_k, c_k) is a message routed between kingdoms C_i undergoing translation T_k in travel while the translator retains memory of the message through the shift operator. The route function R specifies the future destination $R(i_k, c_k)$ of the message c_k given its current location i_k .