Optical Phase Conjugation

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The phase conjugated image of an optical signal is a light beam which has its phase *complex conjugated* with respect to the original signal. A device that can generate such a phase conjugated replica is called a phase conjugator. In order to see the physical significance of the operation of phase conjugation on an optical signal, consider a traveling plane wave with angular frequency ω and wave vector $\vec{k}$. The direction of vector $\vec{k}$ is the propagation direction of the light beam and the dispersion relation is $\omega = c|\vec{k}|$, with c the speed of light, and we assume that the wave travels in vacuum. The electric field E of the wave has the form

$$E(\vec{r},t)_{inc} \propto e^{i(\vec{k}\cdot\vec{r}-\omega t)} + \text{c.c.}, \qquad (1)$$

suppressing the polarization (vector character) and an overall phase and amplitude factor. The subscript 'inc' indicates that this is the incident field on the phase conjugator, and c.c. stands for complex conjugate. The phase of this field is the spatial part $\exp(i\vec{k}\cdot\vec{r})$ in the first term and $\exp(-i\vec{k}\cdot\vec{r})$ in the second term (the c.c.), and the phase conjugated image of this wave is therefore, by definition,

$$E(\vec{r},t)_{pc} \propto e^{i(-\vec{k}\cdot\vec{r}-\omega t)} + \text{c.c.}\,. \qquad (2)$$

Effectively, the result of phase conjugation here is that wave vector $\vec{k}$ is replaced by $-\vec{k}$, and hence this field is again a traveling plane wave with angular frequency ω, but the direction of propagation is reversed, i.e. the conjugated wave counterpropagates the incident wave. This situation is schematically illustrated in Figure 1, and the difference with an ordinary mirror (dielectric) is shown.

Equation 2 can alternatively be written as

$$E(\vec{r},t)_{pc} \propto e^{-i(\vec{k}\cdot\vec{r}-\omega(-t))} + \text{c.c.}\,, \qquad (3)$$

and if we interchange the two terms this becomes

$$E(\vec{r},t)_{pc} \propto e^{i(\vec{k}\cdot\vec{r}-\omega(-t))} + \text{c.c.}\,. \qquad (4)$$

Comparing to Equation 1 then shows that the phase conjugated field is identical in form to the incident field, except that time t is replaced by –t. Therefore, E_{pc} and E_{inc} are related as

$$E(\vec{r},t)_{pc} \propto E(\vec{r},-t)_{inc}\,. \qquad (5)$$

Consequently, the phase conjugated wave is identical to the time reversed image of the incident wave. The term phase conjugation is used throughout western literature, but in Russian literature phase conjugation is usually referred to as time reversal. More complicated light signals, where the wave fronts carry the information, can always be decomposed into plane waves by means of a Fourier analysis. Then phase conjugation of each plane-wave component of the signal effectively yields the time reversed replica of the original signal. As an example, consider the radiation emitted by a point source near the surface of a phase conjugator, as shown in Figure 2. The incident field is a traveling outgoing spherical wave. When this field reflects at a mirror then the reflected field is again a spherical wave, propagating as shown in the figure. But when the same field is reflected at a phase conjugator, then the phase conjugated wave is an incoming spherical wave that travels back to the point source. This can be understood by considering a small part of a wave front as approximately a plane wave, and then constructing the image as in Figure 1. Alternatively, from the fact that the reflected wave is the time reversed image of the incident field, it follows immediately that the conjugated wave has to retrace the path of the incident radiation. This feature of

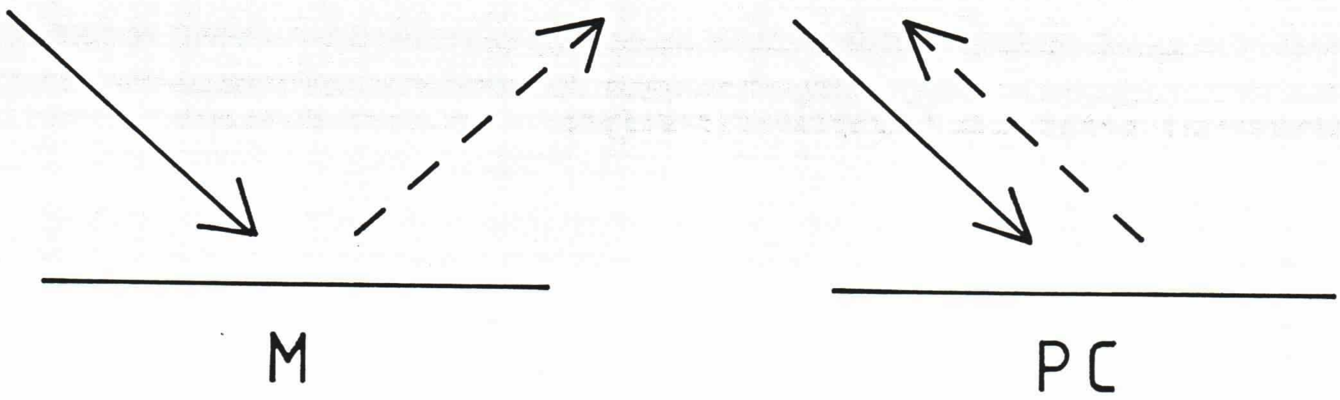

Figure 1. When a light wave is incident (solid arrow) on a mirror (M), then the reflected wave (dashed arrow) propagates into the specular direction, such that the angle of reflection is equal to the angle of incidence. On the other hand, if a wave reflects at a phase conjugator (PC), then the phase-conjugated field counterpropagates the original beam.

a phase conjugator is called self-focusing or self-targeting, for obvious reasons.

The most important practical application of optical phase conjugation appears to be wavefront correction. When an optical wavefront is distorted in, say, passing through some medium, then the distortions can be 'undone' by phase conjugating the signal and letting the wavefront traverse the same medium again. The general idea is illustrated in Figure 3. A planer wavefront passes through the distorter, and is subsequently reflected at a phase conjugator. Since the phase conjugated image is the time-reversed replica of the original wavefront, the distortions will build up in a time reversed manner when the front passes through the distorter for the second time. As a result, the distortions in the emerging wavefront will have disappeared, and the new signal again has a planar

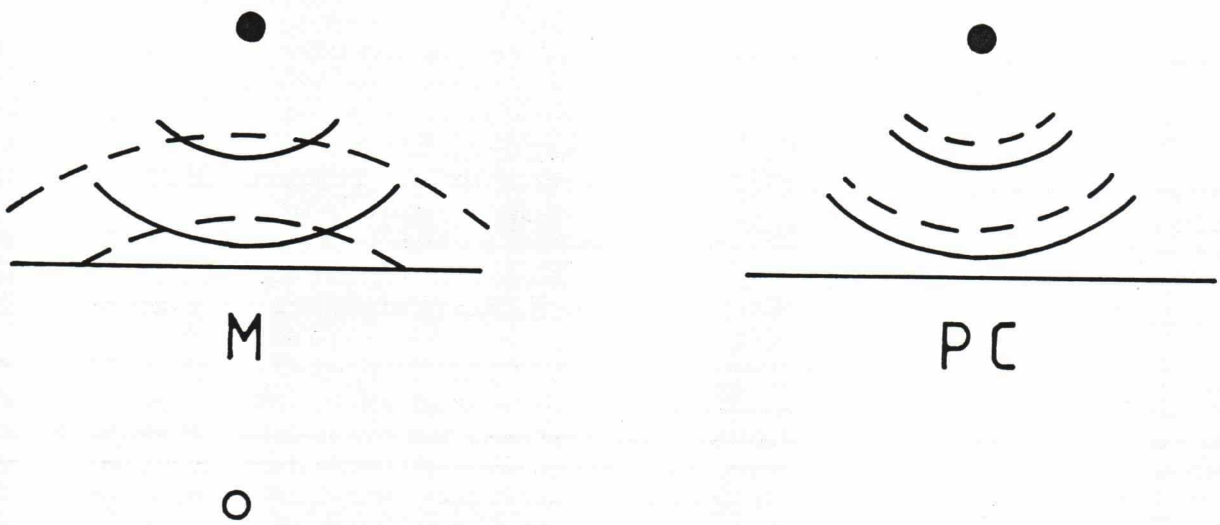

Figure 2. Spherical waves emitted by a point source (solid circle) reflect as outgoing waves at a mirror, and it appears that the waves emanate from a mirror source (open circle) behind the mirror. This image source is located at an equal distance to the mirror as the original source. For reflection at a phase conjugator, however, the reflected field consists of incoming spherical waves that travel back to the source.

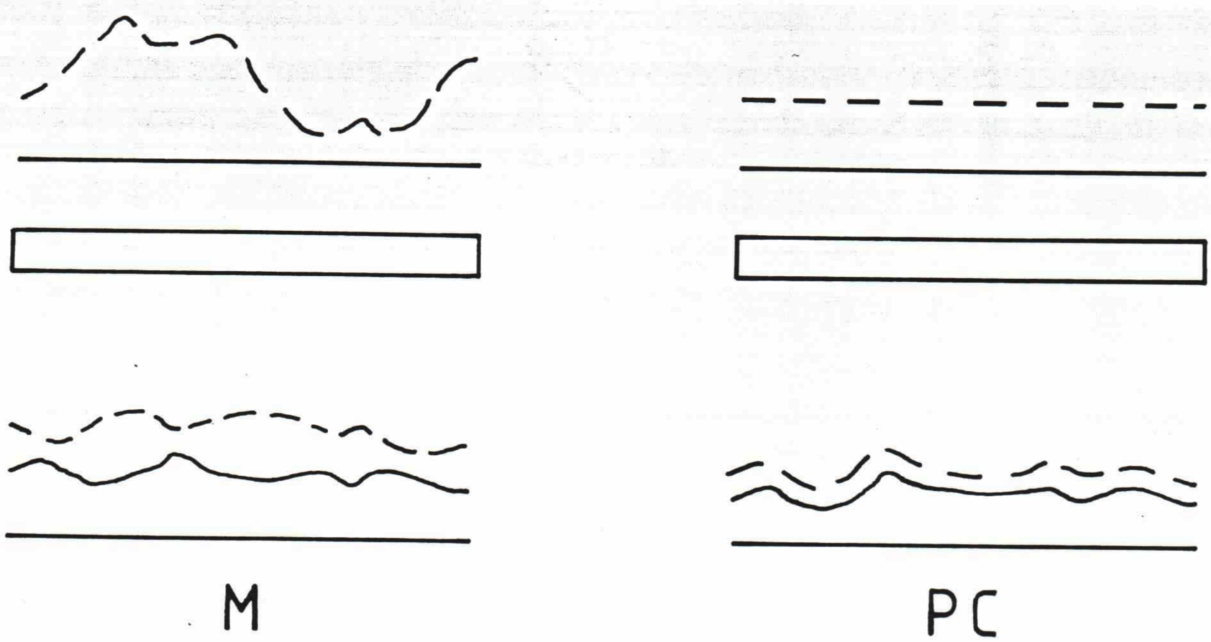

Figure 3. A planar wavefront (solid straight line) passes through a distorter and then reflects at either a mirror or a phase conjugator. Then it passes through the same distorter for a second time. For the configuration with the phase conjugator, the final wavefront is free from distortions (dashed straight line).

wavefront. For comparison, when the phase conjugator would be replaced by an ordinary mirror, then the reflected signal is not the time-reversed image and during the second passage through the medium the distortions will build up again, leaving the wavefront completely distorted. This ability of correcting wavefronts has many applications in optical amplifier systems and lasers.

Wavefront correction through phase conjugation was demonstrated experimentally for the first time in 1972 at the Lebedev Physical Institute in Moscow by Zel'dovich et al. (1). The wavefront of a laser beam was distorted by letting it pass through an etched glass plate. Subsequently, the beam was sent into a cell with methane gas which produced backscattered light via stimulated Brillouin scattering. This light was then sent through the glass plate in the opposite direction, and it appeared that the distortions had disappeared. They concluded that the cell operated as a phase conjugator. After this experimental milestone, the field of optical phase conjugation developed rapidly. Phase conjugation in liquid CS_2 by Brillouin scattering (2) and Raman scattering (3) was demonstrated experimentally, and in 1977 Hellwarth (4) and Yariv and Pepper (5) proposed to construct a phase conjugator based on four-wave mixing in liquids or crystals. The great advantages of that scheme, compared to Brillouin scattering, are that the response time of the medium is negligible, there is no frequency shift with the acoustic frequency, and the required laser power is much less. About a year later, wavefront reversal by four-wave mixing in liquid CS_2 (6,7) and with a lithium formate crystal (8) was realized experimentally. When a photorefractive crystal is used as the medium for four-wave mixing, then, under certain circumstances, there is no need for external pump beams. Such a self-pumped phase conjugator was first observed experimentally by Feinberg (9) with a $BaTiO_3$ crystal.

A possible geometry for a four-wave mixer operating as a phase conjugator is shown in Figure 4. A slab of nonlinear nearly-transparent material is illuminated by two strong counterpropagating laser beams with frequency $\bar{\omega}$, and a much weaker field with frequency ω is incident on the surface of the medium, as shown in the figure. The two pump beams and the incident field couple together through the third-order susceptibility tensor, and this induces a polarization in the medium which is proportional to the pump intensity and to the electric field of the incident radiation. This then leads to the generation of a new signal, which leaves the crystal in a direction opposite to the incident field and which is the phase-conjugated image of the incident field. In addition, there will be a reflected wave in the specular direction due to the linear interaction, and a transmitted wave which leaves the crystal at the other side.

When the frequency ω of the incident field is not equal to the pump frequency $\bar{\omega}$, then the phase conjugated signal will have a frequency of $2\bar{\omega}-\omega$ which is very slightly

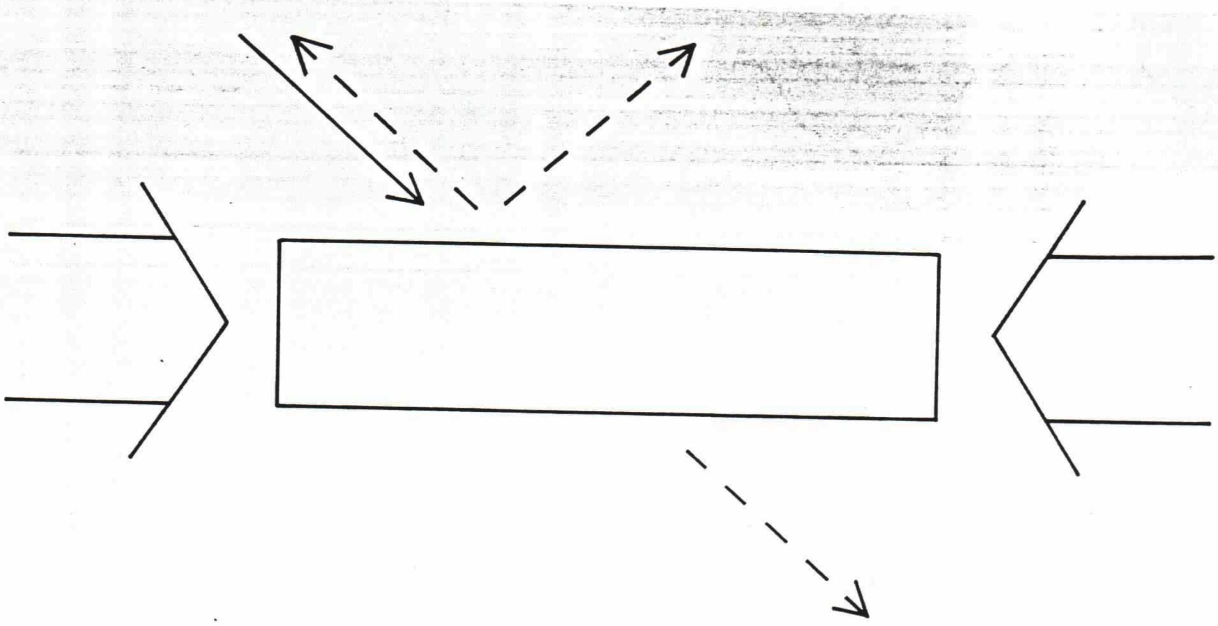

Figure 4. *A layer of nonlinear material is illuminated from two sides by strong laser light (big arrows), and this combination operates as a phase conjugator. A field incident from the top then generates a phase-conjugated image and a specular reflection, and part of the light will be transmitted through the layer.*

shifted. This also yields a small angle between the incident and phase-conjugated waves. If we write E_{pc} and E_{inc} for the amplitudes of both waves, then they are related by

$$E_{pc} = FE_{inc}\,, \tag{6}$$

with F the (complex) Fresnel coefficient. The absolute

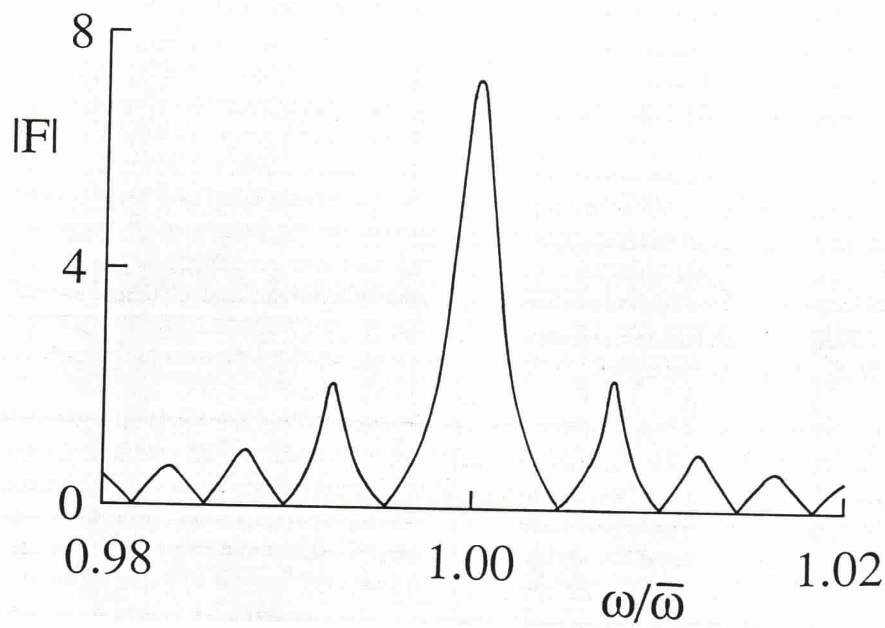

Figure 5. *Typical behavior of the absolute value of the Fresnel reflection coefficient for the phase-conjugated wave. The angular frequency of the incident wave, ω, is normalized with the pump frequency $\bar{\omega}$. This graph is for a surface-polarized incident wave, a transparent crystal (dielectric constant equal to unity), and an angle of incidence equal to 45°.*

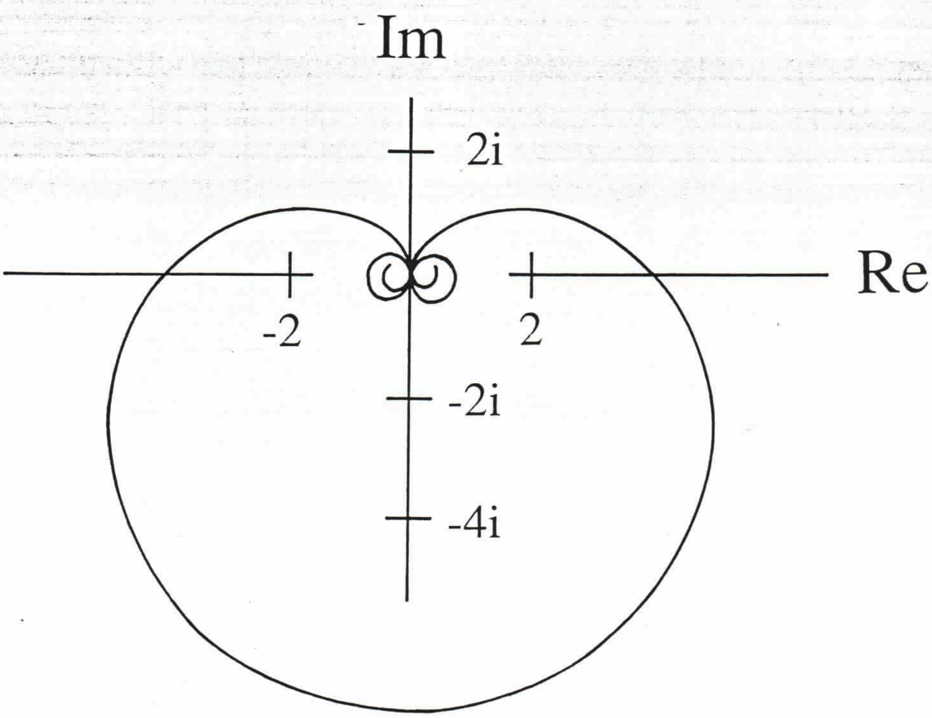

Figure 6. Polar diagram of F as a function of ω in the complex plane for the same parameters as in Figure 5. At $\omega=\bar{\omega}$ the value of F is negative imaginary, corresponding to a phase shift of 90° upon reflection.

value of F is the amplitude ratio of both fields and the argument of F is the phase shift at reflection. The Fresnel coefficient can be calculated explicitly (10), but the expression is very complicated. It depends on the dielectric constant, an element of the susceptibility tensor, the thickness of the layer, the frequency ω relative to $\bar{\omega}$, the angle of incidence, the polarizations of the incident field and the pumps, and the intensity of the pumps. A typical example of |F| as a function of $\omega/\bar{\omega}$ is shown in Figure 5. The Fresnel coefficient has a pronounced maximum for $\omega=\bar{\omega}$. This frequency dependence is entirely geometrical, and is not a consequence of a frequency dependent dielectric constant or susceptibility tensor. Interesting to notice is that |F| can exceed unity, which implies amplification of the signal. Such amplification was predicted within a simplified theory (11), and observed experimentally (12). In principle, |F| can approach infinity, and this phenomenon is called self-oscillation. The required energy for amplification is supplied by the pump beams, and this can lead to pump depletion. In Figure 6 the Fresnel coefficient is represented by a polar diagram in the complex plane, and for the same parameters as in Figure 5. This diagram illustrates the dependence of the phase shift on the frequency. Notice that, on resonance, the Fresnel coefficient is purely imaginary, corresponding to a phase shift of π/2. Finally, Figure 7 gives an example of the behavior of |F| as a function of the angle of incidence.

For an ideal phase conjugator, the Fresnel coefficients should not depend on the angle of incidence, the frequency, or the polarization of the incident wave, and there should not be a shift in frequency nor a change of propagation direction. Such ideal phase conjugators, however, do not exist. It can be shown that the phase conjugator described above preserves the helicity upon reflection, which is a necessary requirement for proper operation of a phase conjugator (13). Many other methods for the generation of the phase-conjugated image of an optical signal exist, each one of them having its own advantages and disadvantages, and specific imperfections. For extensive reviews on the various methods and aspects of phase conjugation we refer to the literature (14–16).

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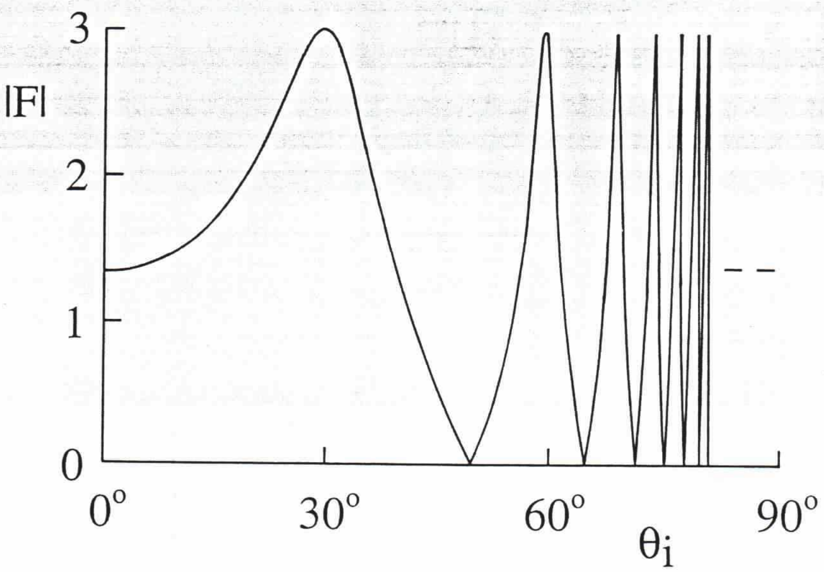

Figure 7. Illustration of the dependence of |F| on the angle of incidence θ_i. For large θ_i the behavior becomes very oscillatory.

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