The Formal Rules of Algebra

Summary of the formal rules of algebra on the set of real numbers

1. The axioms of "equality"

$a = a$	**Reflexive or Identity**
If $a = b$, then $b = a$.	**Symmetry**
If $a = b$ and $b = c$, then $a = c$.	**Transitivity**

These are the "rules" that govern the use of the = sign.

2. The commutative rules of addition and multiplication

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

3. The associative rules of addition and multiplication

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

4. The identity elements of addition and multiplication:

$$a + 0 = 0 + a = a$$

$$a \cdot 1 = 1 \cdot a = a$$

0 and 1are the identity elements for addition and multiplication respectively

5. The additive inverse of *a* is −*a*

$$a + (-a) = -a + a = 0$$

6. The multiplicative inverse or reciprocal of *a is* symbolized as $\frac{1}{a}$ $(a \neq 0)$

$a \times \frac{1}{a} = \frac{a}{a} = 1$, The **product** of **a number** and its **reciprocal** is **1**

Two numbers are called ***reciprocals*** of one another if their product is 1.
1/*a* and *a* are **reciprocal** to each other.
The **reciprocal** of **p/q is q/p.**

7. The algebraic definition of subtraction

$$a - b = a + (-b)$$

Subtraction, in algebra, is defined as ***addition* of the inverse**.

8. The algebraic definition of division

$$a \div b = \frac{a}{b} = a \times \frac{1}{b},\ b \neq 0$$

Division, in algebra, is defined as ***multiplication*** by the reciprocal.
Hence, algebra has two fundamental operations: addition and multiplication.

9. The inverse of the inverse

$$-(-a) = a$$

10. The relationship of $b - a$ *to* $a - b$

$$b - a = -(a - b)$$

11. The Rule of Signs for multiplication, division, and fractions

$$a(-b) = -ab. \quad (-a)b = -ab. \quad (-a)(-b) = ab.$$

$$\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b} \text{ and } \frac{-a}{-b} = \frac{a}{b}$$

Note: "Like signs produce a positive number; unlike signs, a negative number."

12. Rules for 0

$$a \cdot 0 = 0 \cdot a = 0$$

If $a \neq 0$, then $\frac{0}{a} = 0$, but $\frac{a}{0}$ is not defined

13. Multiplying/Factoring

$m(a + b) = ma + mb$ **The distributive rule/ Common factor**

14. The same operation on both sides of an equation

If $a = b$, then $a + c = b + c$

If $a = b$, then $ac = bc$

We may ***add*** the same number to both sides of an equation; we may ***multiply*** both sides by the same number.

15. Change of sign on both sides of an equation

If $-a = b$, **then** $a = -b$.

We may change every sign on both sides of an equation.

16. Change of sign on both sides of an inequality: Change of direction (sense)

If $a < b$, then $-a > -b$.

When we **change the signs** on both sides of an inequality, we must change the direction (sense) of the inequality.

17. The Four Forms of equations corresponding to the Four Operations and their inverses

If $x + a = b$, then $x = b - a$.

If $x - a = b$, then $x = a + b$.

If $ax = b$, then $x = {}^{b}/_{a}$

If ${}^{x}/_{a} = b$, then $x = ab$

18. Change of sense when solving an inequality

If $-ax < b$, then $x > -\frac{b}{a}$

19. Multiplication of fractions

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ and $a \times \frac{c}{d} = \frac{ac}{d}$

20. Division of fractions (Complex fractions)

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$ or equivalently $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

Division is multiplication by the **reciprocal**.

21. Addition/Subtraction of fractions

$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$ Same denominator

$\frac{a}{c} \pm \frac{b}{d} = \frac{ad \pm bc}{cd}$ Different denominators

22. Power and exponents

Let n be a natural number, then $a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n-factors}$

Here, a^n is called **power,** n is called **exponent** and a the

Laws of Exponents

Laws		Examples
1)	$x^1 = x$	$6^1 = 6$
2)	$x^0 = 1$	$7^0 = 1$
3)	$x^{-1} = 1/x$	$4^{-1} = 1/4$
4)	$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
5)	$x^m/x^n = x^{m-n}$	$x^6/x^2 = x^{6-2} = x^4$
6)	$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times 3} = x^6$
7)	$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
8)	$(x/y)^n = x^n/y^n$	$(x/y)^2 = x^2 / y^2$
9)	$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$

And the Laws about Fractional Exponents:

10)	$x^{1/n} = \sqrt[n]{x}$	$x^{1/3} = \sqrt[3]{x}$
11)	$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$	$x^{\frac{2}{3}} = \sqrt[3]{x^2} = \left(\sqrt[3]{x}\right)^2$

Proof of **11):** $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$ follows from the fact that

$$\frac{m}{n} = m \times (1/n) = (1/n) \times m$$