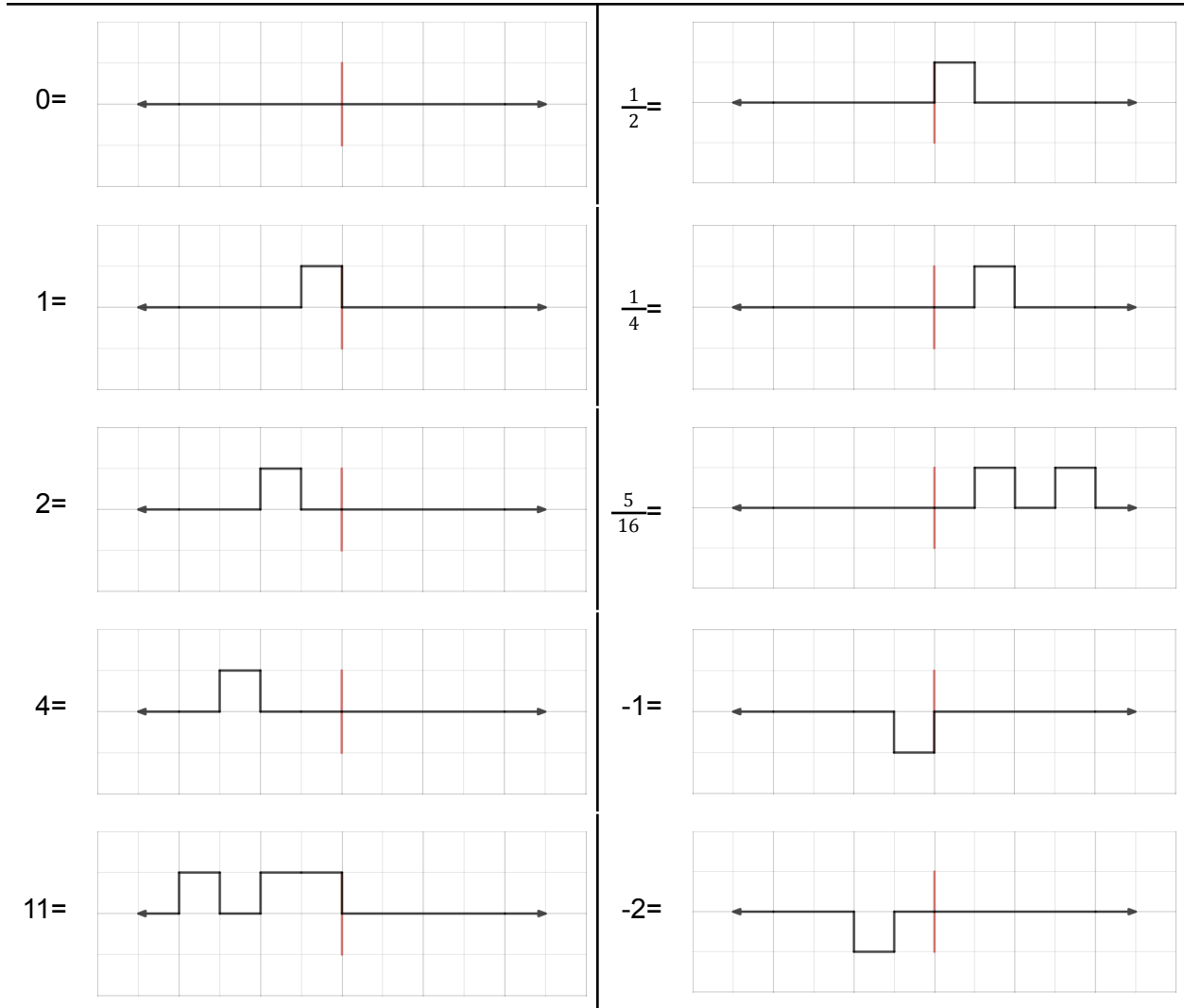
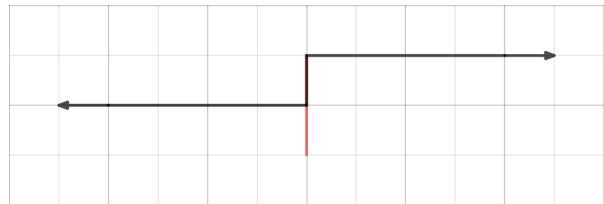


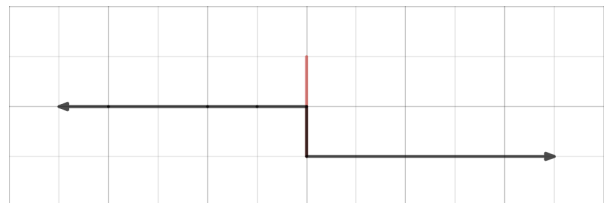
# Line-Forms



$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{n \geq 1} \frac{1}{2}^n = \frac{x}{1-x} \Big|_{\frac{1}{2}} = 1 =$$



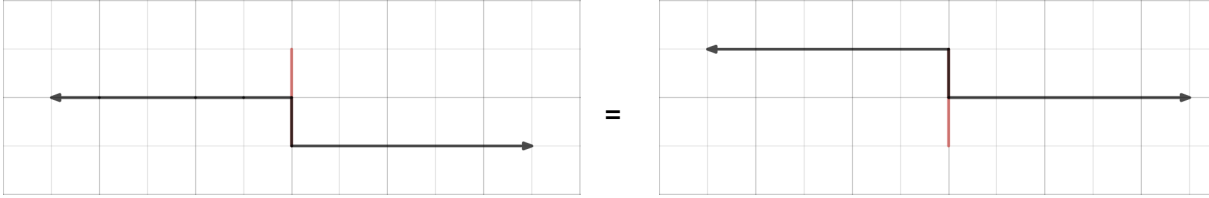
$$\frac{-1}{2} + \frac{-1}{4} + \frac{-1}{8} + \dots = - \sum_{n \geq 1} \frac{1}{2}^n = -1 =$$



**Consider:**

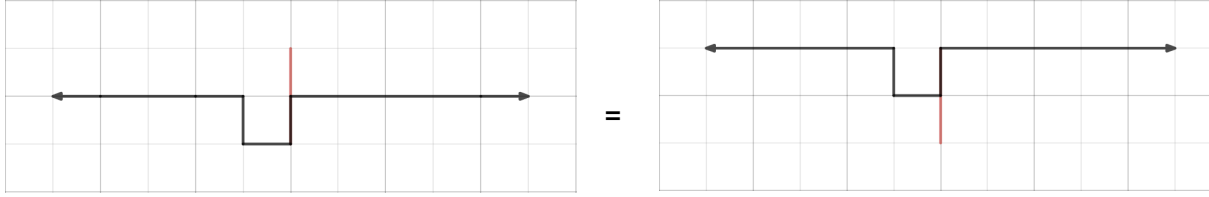
Globally translating the lineform one unit to the left or right corresponds to multiplying by 2 or 1/2 respectively. Locally translating a cell up or down corresponds to adding or subtracting a power of 2. However, global vertical translations have no obvious meaning; they would involve manipulating divergent series of powers of 2.

Assume global vertical translation has no effect. Then, all global vertical translations of a given lineform are equal. So:



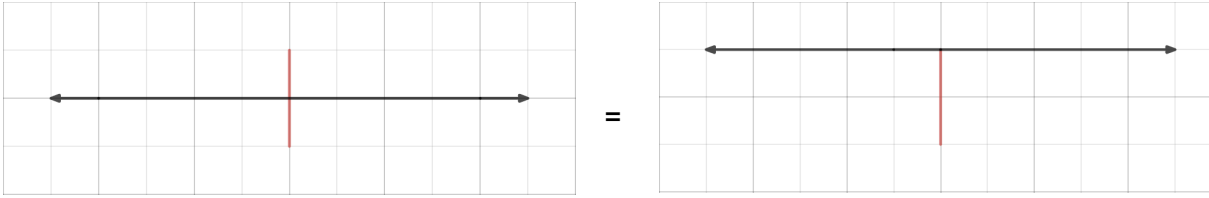
$$\frac{-1}{2} + \frac{-1}{4} + \frac{-1}{8} + \dots = - \sum_{n \geq 1} \frac{1}{2^n} = \frac{-x}{1-x} \Big|_{\frac{1}{2}} = -1 = \frac{1}{1-x} \Big|_2 = \sum_{n \geq 0} 2^n = \dots + 8 + 4 + 2 + 1$$

Likewise:



$$\dots + 0(2^2) + 0(2^1) - 2^0 + \frac{0}{2} + \frac{0}{4} + \dots = -1 = \dots + 2^2 + 2^1 + 0(2^0) + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

Interestingly:



$$\dots + 0(2^1) + 0(2^0) + \frac{0}{2} + \dots = - \sum_{-\infty}^{\infty} 0^n = 0 = \sum_{-\infty}^{\infty} 2^n = \dots + 2^2 + 2^1 + 2^1 + \frac{1}{2} + \frac{1}{4} + \dots$$